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ARITHMETIC.

I.—NOTATION AND NUMERATION.

ARITHMETIC teaches how to calculate or reckon by means of numbers.

Numbers are expressed by means of **figures** or **digits.**

The art of *writing* a number in figures is called **Notation.**

The art of reading, or expressing in words a number written in figures, is called **Numeration.**

The figures, or digits, used in Arithmetic, are :—

0, 1, 2, 3, 4, 5, 6, 7, 8, 9;

which stand for *nought* or *nothing*, *one*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine*.

The sign 0 (*nought*, *cipher*, or *zero*) signifies nothing by itself, but it serves to fill up vacant places in a line of figures, and when placed on the *right hand* of other figures it increases their value *tenfold.*

The *local* value of any figure depends on the place which it occupies in the line.

A figure standing by *itself*, or on the *right hand* of a line of figures, expresses so many **units** or ones.

A figure standing in the *second* place from the right, expresses so many **tens** of units; in the *third* place, so many *tens of tens*, or **hundreds** of units; in the *fourth* place, so many *tens of hundreds*, or **thousands** of units; and so on, according to the following table, each figure increasing tenfold in value as it advances from right to left.

This tenfold increase in the local value of a figure has caused our system or scale of notation to be called the Decimal system.

NUMERATION TABLE.

7	8	9	1	2	3	4	5	6	7	8	9
hundreds of thousands of millions.	tens of thousands of millions.	thousands of millions.	hundreds of millions.	tens of millions.	millions.	hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.

The above number is to be read, seven hundred and eighty-nine thousand, one hundred and twenty-three millions, four hundred and fifty-six thousand, seven hundred and eighty-nine.

For the more easily numerating a long line of figures, it is usual to divide it into periods of six, or half-periods of three figures, beginning at the right.

The *first* period of six being *units;* the *second* period, *millions;* the *third* period, *billions;* the *fourth* period, *trillions;* and so on to *quadrillions*, *quintillions*, *&c.* A series of figures, containing more than three half-periods, *i. e.* nine figures, is, however, very seldom required.

The left hand figure in each half-period, as in the above table, occupies the place of *hundreds*, *i. e.* of *hundreds* simply, *hundreds* of thousands, *hundreds* of millions, &c.

Ex. 1.

Write the following numbers in WORDS :

(1.) 34
(2.) 97
(3.) 601
(4.) 869
(5.) 650
(6.) 5794
(7.) 4909
(8.) 6717
(9.) 89028
(10.) 30102
(11.) 76029
(12.) 827171
(13.) 7546738
(14.) 5931081
(15.) 17643000
(16.) 80970503
(17.) 309025607
(18.) 980076011

19. By the census of 1861 the number of persons in England and Wales was 20,061,725; in Scotland, 3,061,117; in Ireland, 5,764,543; in the Channel Islands and the Isle of Man, 143,779; making together, 29,031,164.

20. The following are the mean distances of the planets from the sun, in miles :—

Mercury	35,551,710
Venus	66,431,450
Earth	91,841,100
Mars	139,937,400
Jupiter	477,831,000
Saturn	876,058,000
Urănus	1,761,763,000
Neptune	2,758,566,000

Ex. 2.

Write the following numbers in FIGURES.

(1.) Thirteen; seventeen; twelve; twenty-one; forty-nine.

(2.) One hundred and twenty-eight; three hundred and thirteen.

(3.) Two hundred and seventy-six; four thousand and four.

(4.) Twelve hundred and eleven; thirteen hundred and ninety.

(5.) Twenty-eight thousand, three hundred and nineteen.

(6.) Three millions, one hundred and forty-five thousand, six hundred and eighty-five.

(7.) Eighty-six millions, four hundred and three thousand and seven.

(8.) Seven hundred and thirty millions, ninety thousand and fifteen.

(9.) Nine hundred and nine millions, ninety-nine thousand and nine.

(10.) Eleven hundred and eleven millions, eleven thousand one hundred and eleven.

(11.) The population of London, in eighteen hundred and sixty-one, was two millions, eight hundred and four thousand.

(12.) One million, two hundred and thirty thousand, eight hundred and ninety-six natives of Ireland, and eight hundred and twenty-three thousand, eight hundred and thirty-seven natives of Great Britain emigrated between the years 1851 and 1861.

ROMAN NUMERALS.

The ancient Romans made use of seven capital letters to express numbers, viz. :—

I.	V.	X.	L.	C.	D. or IↃ.	M. or CIↃ.
1.	5.	10.	50.	100.	500.	1000.

and by combining these, all other numbers were represented.

When a letter of lower or equal value is placed to *the right* of another, the value is *increased;* thus, XI stands for eleven, that is, ten *and* one; XX, two tens or twenty; DC, six hundred.

When a letter of lower value is placed to *the left* of another, the value is *decreased;* thus, IV stands for four, that is, five *less* one; XL, fifty *less* ten, or forty.

Ex. 3.

Express in words or figures the following numbers :—

(1.) XII, XXX, XXV, LXVI, LXXXVIII, CCCXXVII, MDLV.

(2.) IX, XIV, XLIX, XC, CXCIII, MDXLV, MDCLXIX.

(3.) MDCCCLVII, MDXC, CXIX, MDCCLXXXIX, MDCCCLXV.

Write the following in Roman numerals :—

(4.) The numbers from one to a hundred.

(5.) 115, 276, 501, 607, 1066, 1265, 1314, 1415, 1666, 1792, 1815.

(6.) The dates of the accession of the sovereigns of England since the Norman conquest.

II.—ADDITION.

Addition is the collecting or putting together of two or more numbers to find their **sum** or **total.**

The sign of Addition is + (*plus*, more), and shows that the numbers between which it is placed are to be added together; thus, 2 + 4 = 6, that is, two *plus* four *equals* six; 7 + 5 = 12, or seven *and* five *are* twelve; and so on.

ADDITION TABLE.

2 and 1 are 3	4 and 7 are 11	7 and 1 are 8	9 and 7 are 16
2 — 2 — 4	4 — 8 — 12	7 — 2 — 9	9 — 8 — 17
2 — 3 — 5	4 — 9 — 13	7 — 3 — 10	9 — 9 — 18
2 — 4 — 6	4 — 10 — 14	7 — 4 — 11	9 — 10 — 19
2 — 5 — 7	4 — 11 — 15	7 — 5 — 12	9 — 11 — 20
2 — 6 — 8	4 — 12 — 16	7 — 6 — 13	9 — 12 — 21
2 — 7 — 9	5 and 1 are 6	7 — 7 — 14	11 and 1 are 12
2 — 8 — 10	5 — 2 — 7	7 — 8 — 15	11 — 2 — 13
2 — 9 — 11	5 — 3 — 8	7 — 9 — 16	11 — 3 — 14
2 — 10 — 12	5 — 4 — 9	7 — 10 — 17	11 — 4 — 15
2 — 11 — 13	5 — 5 — 10	7 — 11 — 18	11 — 5 — 16
2 — 12 — 14	5 — 6 — 11	7 — 12 — 19	11 — 6 — 17
3 and 1 are 4	5 — 7 — 12	8 and 1 are 9	11 — 7 — 18
3 — 2 — 5	5 — 8 — 13	8 — 2 — 10	11 — 8 — 19
3 — 3 — 6	5 — 9 — 14	8 — 3 — 11	11 — 9 — 20
3 — 4 — 7	5 — 10 — 15	8 — 4 — 12	11 — 10 — 21
3 — 5 — 8	5 — 11 — 16	8 — 5 — 13	11 — 11 — 22
3 — 6 — 9	5 — 12 — 17	8 — 6 — 14	11 — 12 — 23
3 — 7 — 10	6 and 1 are 7	8 — 7 — 15	12 and 1 are 13
3 — 8 — 11	6 — 2 — 8	8 — 8 — 16	12 — 2 — 14
3 — 9 — 12	6 — 3 — 9	8 — 9 — 17	12 — 3 — 15
3 — 10 — 13	6 — 4 — 10	8 — 10 — 18	12 — 4 — 16
3 — 11 — 14	6 — 5 — 11	8 — 11 — 19	12 — 5 — 17
3 — 12 — 15	6 — 6 — 12	8 — 12 — 20	12 — 6 — 18
4 and 1 are 5	6 — 7 — 13	9 and 1 are 10	12 — 7 — 19
4 — 2 — 6	6 — 8 — 14	9 — 2 — 11	12 — 8 — 20
4 — 3 — 7	6 — 9 — 15	9 — 3 — 12	12 — 9 — 21
4 — 4 — 8	6 — 10 — 16	9 — 4 — 13	12 — 10 — 22
4 — 5 — 9	6 — 11 — 17	9 — 5 — 14	12 — 11 — 23
4 — 6 — 10	6 — 12 — 18	9 — 6 — 15	12 — 12 — 24

RULE.—Place the numbers under one another, beginning at the right hand, so that the units may be under units, the tens under tens, &c., and draw a line under them.

Add the figures in the right hand column, and if they come to *more than nine,* write down the *unit* figure, and *carry* the ten, or tens, to the next column; then add the figures in the other columns in the same manner, and in the last, or left hand column write down the *whole* amount.

EXAMPLE.—7325
8772
4941
9544
5962
———
36544

Worked thus: 2 + 4 + 1 + 2 + 5 = 14; put down 4 and carry 1 to the *tens* column; because 14 = one ten and 4 units.

Then 1 + 6 + 4 + 4 + 7 + 2 = 24; put down 4 and carry 2 to the *hundreds'* column (because 24 tens = 2 hundreds and 4 tens).

Then 2 + 9 + 5 + 9 + 7 + 3 = 35; put down 5 and carry 3 to the *thousands'* column (because 35 hundreds = 3 thousands and 5 hundreds).

Lastly, 3 + 5 + 9 + 4 + 8 + 7 = 36; put down 36 (thousands)

PROOF.—The correctness of an addition sum may be proved by adding the numbers *downwards*, when the total will be the same as they were when added upwards.

Ex. 4.

(1.)	3426	(2.)	45673	(3.)	763241	(4.)	234567
	9024		89014		451907		890235
	5106		23455		639015		109876
(5.)	15897	(6.)	90423	(7.)	31141	(8.)	13579
	68561		78514		25639		24680
	32426		56186		48507		69154
	59791		34975		29849		37812
	43043		12639		67635		95643
(9.)	123456	(10.)	314159	(11.)	793253	(12.)	867867
	769769		824568		867547		732357
	486756		963963		324687		797496
	762937		896735		326593		963979
	893893		732467		893748		816598
(13.)	545687	(14.)	509267	(15.)	838	(16.)	457186
	369454		235809		4298		574304
	732574		72920		50196		1031490
	743173		8392		730205		2062980
	839768		429		9180634		4125960

(17.) 64738
28674
35978
67536
93625
28967

(18.) 328695
847598
473876
873569
695724
958293

(19.) 916738
892654
491673
389265
549167
738926

(20.) 67891
34567
102368
89128
45678
91234

(21.) 562347
85476
36728
6544
97328
475564

(22.) 347326
587765
768246
87335
654
363578

(23.) 876208
343672
394625
4713
29875
368207

(24.) 6752
784519
22687
456275
44824
612785

(25.) 478
614562
9875
589
423765
48567

(26.) 6817
695737
462385
752
93607
342665

(27.) 682475
468854
713628
299
51876
2945

(28.) 371628
85738
654297
8277
365685
423748

(29.) 896356
729389
674869
643387
869643
323232

(30.) 823676
767823
476467
696894
568969
467935

(31.) 896567
269846
673734
893382
324742
742428

(32.) 968497
732456
123759
567459
732684
684567

(33.) 73846
29873
48765
38214
47386
96786
78738

(34.) 8749638
6859639
6916387
3878964
8738495
4856877
5293750

(35.) 89417675
714877
7891466
815677
77361648
58329412
23798657

(36.) 673673
968759
123456
567459
732567
684497
732684

(37.)	(38.)	(39.)	(40.)
72473682	107485075	5962847	5498
74928	870302861	5196382	672457
5536	73628874	324	786257
674976	668279	648900	35396284
7967450	47538234	8007119	85936428
13207895	267849026	9127644	97894
34827583	486792836	8272491	18769662
27590328	9563748	6378948	91731234
56385517	6473928	4677836	85373948

(41.) 314159 + 767 + 823548 + 3927 + 367367 + 99999 + 873873.

(42.) 768357 + 4938 + 392739 + 666 + 746328 + 6969 + 74747.

(43.) 9896 + 77693 + 4692 + 683 + 6693.

(44.) 3216745 + 291674 + 24367 + 3467 + 76723.

(45.) 29 + 8764 + 42734 + 473 + 97354 + 89673 + 2768 + 21847 + 47385 + 7698.

(46.) 239803 + 4668231 + 2101104 + 1345702 + 6143623 + 4755635 + 93726 + 52794 + 18964 + 1626 + 377980 + 5800 + 9998986 + 705921 + 1700000.

(47.) Three hundred and three thousand, five hundred and sixty-eight + one hundred and ninety-five thousand, four hundred and ninety-two + three hundred and ninety thousand, nine hundred and ninety-seven + fifty-eight thousand, two hundred and eighty-seven + two millions, and thirty-one thousand, two hundred and thirty-six + one million, seven hundred and ninety-seven thousand, nine hundred and ninety-five.

(48.) Twenty-seven thousand and eight + three thousand and nineteen + seventy-seven thousand and ten + eight hundred and seventy-three + fifty thousand, nine hundred and seven + twelve thousand, two hundred and ninety-six + eighty thousand + eighteen hundred and seventeen.

(49.) Forty thousand and four + six thousand and sixty + ten millions and one + ten thousand three hundred and eleven + five hundred and five thousand, three hundred and four + sixty-seven thousand and nine + eight millions, sixty thousand and twenty-five.

(50.) Seven millions, four hundred and seven thousand and three +

one hundred and sixty-nine thousand, two hundred and five + three hundred and eighty-two + four thousand eight hundred and fifteen + one million, one hundred thousand, one hundred and one + twenty thousand and sixty-one.

(51.) The Duke of Wellington's army at Waterloo was composed of:—Infantry, twenty thousand six hundred and sixty-one; Cavalry, eight thousand seven hundred and thirty-five; Artillery, six thousand eight hundred and seventy-seven: besides thirty-three thousand four hundred and thirteen Belgian and other foreign allies. What was the whole number of his army?

(52.) Napoleon's army at Waterloo was composed of:—Infantry, forty-eight thousand nine hundred and fifty; Cavalry, fifteen thousand seven hundred and sixty-five; and Artillery, seven thousand seven hundred and thirty-two. How many Frenchmen were engaged in the battle?

(53.) In 1841, 1851, and 1861, the population returns were as follows:—

	1841.	**1851.**	**1861.**
England and Wales ...	15,911,725	17,922,768	20,061,725
Scotland..................	2,628,957	2,870,784	3,061,329
Ireland	8,205,382	6,515,794	5,764,343
Channel Islands, &c ...	124,079	143,126	143,779

Find the total population at each period.

(54.) The following was the number of houses in England and Wales in 1851 and 1861. Find the total in each year.

	1851.	**1861.**
Inhabited............	3,278,039	3,745,463
Uninhabited	153,494	182,325
Building	26,571	27,580

(55.) Find the population of the six northern counties of England in 1851 and 1861.

	1851.	**1861.**
Northumberland ...	303,568	343,028
Cumberland	195,492	205,293
Durham	390,997	509,018
Westmoreland	58,287	60,809
Lancashire............	2,031,236	2,428,744
Yorkshire	1,797,995	2,033,051

(56.) Find the total number of pounds of cotton imported into the United Kingdom during 1860.

From United States	1,115,890,608
„ Brazil	17,286,864
„ Egypt	43,954,064
„ East Indies	204,132,208
„ Other countries	9,666,048

III.—SUBTRACTION.

Subtraction is the taking of one number from another, in order to find the **difference** between them.

The sign of Subtraction is — (*minus*, less), and shows that the number to the right of it is to be subtracted; thus, 12 — 7 = 5, that is, twelve *minus* seven *equals* five.

The greater of the given numbers is called the **minuend**, the less the **subtrahend**.

Rule.—Place the less number under the greater, so that units may be under units, tens under tens, &c., and draw a line under it.

Begin at the right hand, and take each number in the lower line from the one above it, and set down the remainder under it.

Whenever the lower number is greater than the upper, add ten to the upper, and then subtract the lower as before.

The ten so *borrowed* is to be repaid by *carrying*, or adding *one* to the next figure to the left, in the lower line.

Example.—9659 **Minuend.**
3796 **Subtrahend.**
5863 **Difference**, or **Remainder.**

Worked thus: Six (units) from *nine* (units) leave *three* (units); *put down* 3 in the units' place.

Nine (tens) from five (tens) you cannot, so borrow *ten* (ten tens, or one hundred), and add to the *five;* and say, *nine* (tens) from *fifteen* (tens) leave *six* (tens); *put down* 6 in the tens' place, and *carry* 1 *(that is, the one hundred, or the ten tens borrowed)* to the next figure, which will then be eight.

Eight (hundreds) from *six* (hundreds) you cannot, so borrow *ten* (ten hundreds, or one thousand), and add to the *six;* and say, *eight* (hundreds) from *sixteen* (hundreds) leave *eight* (hundreds); *put down* 8 in the hundreds' place, and *carry* 1 (that is, the one thousand, or ten hundreds borrowed) to the next figure, which will then become four

Four (thousands) from *nine* (thousands) leave *five* (thousands); *put down* 5 in the thousands' place.

PROOF.—The correctness of the subtraction may we proved by adding the remainder to the number subtracted, when the result ought to be the same as the other number.

Ex. 5.

(1.) 531809 507918	(2.) 7020974 2766802	(3.) 5386427 2164315	(4.) 1234567 702973
(5.) 8503402 574271	(6.) 1001251 300043	(7.) 7943027 3428014	(8.) 4500949 999093
(9.) 7767401 6801827	(10.) 2093646 1866476	(11.) 1098863 221119	(12.) 2264684 1460785
(13.) 7559218 6953296	(14.) 3745463 3278039	(15.) 3061329 1447015	(16.) 5764543 2804961
(17.) 6552385 3764543	(18.) 4902735 837596	(19.) 3729568 1982873	(20.) 7502374 1924737

(21.) 2307936854 — 91136537; 80012345 — 432987.

(22.) 3147253678 — 693758269; 67125040 — 8434765.

(23.) 35970462481 — 9738427658; 10010203 — 431859.

(24.) 1004728569362 — 37946259738; 910310 — 456278.

(25.) 987654321 — 123456789; 8057130600 — 148112354.

(26.) 1091 — 176 + 4096; 2736 — 435 + 3728.

(27.) 476 — 91 + 1002 — 569 + 365; 7648 + 36 — 959 + 67 — 3412.

(28.) 527 — 87 + 3361 — 219 + 402 — 3984; 1758 — 799 + 2705.

(29.) What is the difference between 6594689778 and 7648793494?

(30.) How much is 76580074 less than 90019901?

(31.) How much does 1001001 exceed 800056?

(32.) From one hundred and eighty-six thousand, four hundred and sixty-three, take one hundred and twenty thousand and four.

(33.) From eight hundred and sixty thousand, and eighty-six, take seven thousand nine hundred and twenty-nine.

(34.) What is the difference between one million ten thousand and one and one hundred thousand one hundred and nine?

(35.) How much is seven millions, one hundred and eighty-eight less than twelve millions, two hundred and nine thousand and one?

(36.) How much is five millions, thirty thousand and one less than nine millions, one hundred and four thousand and seventeen?

(37.) A gentleman left by his will £11,968 between his son and daughter. The daughter received £5349. What was the son's share?

(38.) The public revenue for the year ending March 31, 1864, was £73,455,430, and the expenditure £71,102,752. What was the surplus?

(39.) A person borrowed £1781, and paid £367; borrowed £1004, paid £1206; borrowed £57, and paid £318. How much does he still owe?

(40.) Of the 20,061,725 persons in England and Wales at the census in 1861, 9,758,852 were males. How many females were there?

(41.) The population of London was 2,362,236 in 1851, and 2,803,034 in 1861. What was the increase in the interval?

(42.) Find the increase or decrease (*a*) in the population of England and Wales, &c.; (*b*) in the number of houses; (*c*) and in the population of the six northern counties (as given in Examples 53, 54, 55, of Addition) between 1851 and 1861.

(43.) A brewer's vat held one thousand and one gallons of beer. Four hundred and ninety-nine gallons were drawn off, and then three hundred and forty-seven gallons more were added. How much beer was now in the vat?

(44.) What sum, taken from a million, will leave six hundred and eighty-nine thousand and eighty-seven? What sum, added to seven hundred and forty-five, will make a thousand.

(45.) Two trains start from the opposite ends of the line between London and Edinburgh, four hundred and twenty miles apart. After one has gone a hundred and eighty-nine miles, and the other a hundred and ninety-eight, how far will they be asunder?

(46.) From the sum of 8,760,432,964 and 5,769,087,579 subtract their difference.

(47.)Take the sum of 12 936, 17,832, and 5094, from the sum of 20,968, 35,798, 21,531, and 6190.

(48.) A tradesman has in his shop goods worth £564, and £25 in his cash box: he owes £334, and debts to the amount of £459 are due to him: what is he worth?

(49.) Mount Everest, the highest mountain in Asia, is 29,002 feet high, and Mont Blanc, the highest in Europe, is 15,732 feet. By how much does the height of the one exceed that of the other?

(50.) The mean distance of Mercury from the Sun is 35,551,710 miles, that of Venus 66,431,450, and that of the Earth 91,841,100 miles; find the distance from the orbit of Mercury to that of Venus; and from the orbit of Venus to that of the Earth.

IV.—MULTIPLICATION.

MULTIPLICATION is the method of finding the amount of a number when repeated a given number of times.

The number *to be multiplied* or repeated is called the **Multiplicand**; the number *by which we multiply*, or the number of repetitions is called the **Multiplier**; and the result or answer is called the **Product.**

The sign of Multiplication is × (multiplied by) and shows that the numbers between which it is placed are to be multiplied together; thus $4 \times 3 = 12$, that is, Four *multiplied by* three *equals* twelve; or, Four *times* three *are* twelve.

Multiplication is in fact a short method of addition; thus, the product of eight times nine, or seventy-two, ($8 \times 9 = 72$), is the same as the amount obtained by adding eight nines or nine eights; $8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 = 72$.

The numbers multiplied together to make any product are called the **Factors** (or makers) of that product; thus 9 and 3 are *the factors* of 27 for $9 \times 3 = 27$.

When three or more numbers are multiplied together the result is sometimes termed the **Continued Product.**

MULTIPLICATION TABLE.

Twice	3 times	4 times	5 times	6 times	7 times	8 times	9 times	10 times	11 times	12 times
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7	1 are 8	1 are 9	1 are 10	1 are 11	1 are 12
2—4	2—6	2—8	2—10	2—12	2—14	2—16	2—18	2—20	2—22	2—24
3—6	3—9	3—12	3—15	3—18	3—21	3—24	3—27	3—30	3—33	3—36
4—8	4—12	4—16	4—20	4—24	4—28	4—32	4—36	4—40	4—44	4—48
5—10	5—15	5—20	5—25	5—30	5—35	5—40	5—45	5—50	5—55	5—60
6—12	6—18	6—24	6—30	6—36	6—42	6—48	6—54	6—60	6—66	6—72
7—14	7—21	7—28	7—35	7—42	7—49	7—56	7—63	7—70	7—77	7—84
8—16	8—24	8—32	8—40	8—48	8—56	8—64	8—72	8—80	8—88	8—96
9—18	9—27	9—36	9—45	9—54	9—63	9—72	9—81	9—90	9—99	9—108
10—20	10—30	10—40	10—50	10—60	10—70	10—80	10—90	10—100	10—110	10—120
11—22	11—33	11—44	11—55	11—66	11—77	11—88	11—99	11—110	11—121	11—132
12—24	12—36	12—48	12—60	12—72	12—84	12—96	12—108	12—120	12—132	12—144

1. *When the multiplier is not greater than* 12.

Rule.—Set the multiplier under the multiplicand, *units* under *units.*

Then, beginning at the right hand, multiply each figure in the multiplicand by the multiplier and set down the product under that figure, remembering that whenever the product is a number of *more than one* figure, *only the right hand figure* is to be set down and *the others are to be carried* to the next product.

EXAMPLE.—7654 **Multiplicand.**
8 **Multiplier.**
61232 **Product.**

Worked thus :— *Eight* times *four* (units) are *thirty-two* (units); put down 2 (units) and carry 3 (tens).

Eight times *five* (tens) are *forty* (tens) and *three* (tens) carried make *forty-three ;* put down three (tens) and carry four (hundreds).

Eight times *six* (hundreds) are *forty-eight* (hundreds) and *four* (hundreds) carried make *fifty-two* (hundreds) ; put down 2 (hundreds) and carry 5 (thousands).

Eight times *seven* (thousands) are *fifty-six* (thousands) and *five* (thousands) carried make *sixty-one* (thousands) ; put down 61 thousands.

If the figures are put down with their *local* value, the operation will stand thus :

$$\begin{array}{rcrcr}
4 & \times & 8 & = & 32 \\
50 & \times & 8 & = & 400 \\
600 & \times & 8 & = & 4800 \\
7000 & \times & 8 & = & 56000 \\
\hline
7654 & & & & 61232
\end{array}$$

When there are ciphers at the *right hand* of either multiplier or multiplicand, set down all the ciphers and then multiply as before.

Ex. 6.

(1). 6196 2	(2.) 2768 3	(3.) 2076 4	(4.) 3542 5
(5.) 27368 5	(6.) 90846 6	(7.) 68973 7	(8.) 936428 7
(9.) 566832 8	(10.) 274629 9	(11) 764324 7	(12.) 897636 9

(13.) Multiply 47475296 by 2, 3, 4, 5, 6.
(14.) „ 334567890 „ 2, 3, 4, 5, 6, 7.
(15.) „ 426938508 „ 3, 4, 5, 6, 7, 8.
(16.) „ 537284161 „ 4, 5, 6, 7, 8, 9, 10.
(17.) „ 818647362 „ 2, 3, 4, 5, 6, 7, 8, 9.
(18.) „ 3764235 „ 7, 8, 9, 11, 12.
(19.) „ 4923567 „ 7, 8, 9, 11, 12.
(20.) „ 7934215 „ 7, 8, 9, 11, 12.
(21.) 6543201 × 4 × 5 × 6 × 7; also by 3 × 7 × 8.
(22.) 987654 × 7 × 8 × 9 × 11; „ „ 5 × 6 × 12.
(23.) 792346 × 7 × 8 × 9 × 11; „ „ 7 × 9 × 12.
(24.) 569745 × 6 × 9 × 11 × 12; „ „ 7 × 8 × 11.

2. *When the multiplier is greater than* 12.

Rule.—Set the multiplier under the multiplicand, units under units.

Then, beginning at the right hand, multiply each figure in the multiplicand by the units' figure, as before.

Next, multiply by the tens' figure and place the right hand figure in the tens' place, that is under the figure you multiply by.

Proceed in the same way with the hundreds, &c., remembering to put the right hand figure of every product under the figure multiplied by.

Lastly, add all the products together to find the whole product.

Example:—Multiply 6196 by 3542.

$$\begin{array}{r}
6196 \\
3542 \\
\hline
12392 = 6196 \times 2 \\
24784\mathit{0} = 6196 \times 40 \\
30980\mathit{00} = 6196 \times 500 \\
18588\mathit{000} = 6196 \times 3000 \\
\hline
21946232 = 6196 \times 3542 \\
\hline
\end{array}$$

The *ciphers in Italics* are usually omitted for convenience.

The **Square** of a number is the number multiplied by itself.

The **Cube** of a number is the number multiplied by itself twice.

Ex. 7.

(1.) 495108 × 17.
(2.) 764074 × 29.
(3.) 478567 × 47.
(4.) 784689 × 72.
(5.) 807650 × 57.
(6.) 908067 × 65.
(7.) 217455 × 68.
(8.) 478109 × 83.
(9.) 765658 × 97.
(10.) 896507 × 67.
(11.) 923169 × 98.
(12.) 947065 × 79.
(13.) 67235987 × 47, 98, 53.
(14.) 43670219 × 94, 65, 87.
(15.) 20314687 × 37, 46, 58.
(16.) 90674839 × 39, 57, 86.
(17.) 6789456 × 407, 829.
(18.) 7896384 × 305, 672.
(19.) 2193769 × 793, 812.
(20.) 7963151 × 876, 905.
(21.) 1275353 × 5467.
(22.) 1174691 × 2763.
(23.) 1756459 × 3947.
(24.) 1020572 × 6793.
(25.) 8246783 × 8769.
(26.) 4526837 × 6857.
(27.) 141884 × 45098.
(28.) 102548 × 45672.
(29.) 861070 × 79369.
(30.) 443520 × 82799.

Ex. 8.

Find the squares of
(1.) 32, 65, 56.
(2.) 371, 225, 634.
(3.) 2345, 3146.
(4.) 3353, 4563.
(5.) 82467, 79674.
(6.) 98763, 49367.

Find the cubes of
(7.) 15, 31, 48.
(8.) 67, 411.
(9.) 203, 638.
(10.) 457, 975.
(11.) 5694, 6893.
(12.) 56947, 79245.

Find the continued products of
(13.) 67 × 285 × 763.
(14.) 623 × 765 × 219.
(15.) 82 × 369 × 471.
(16.) 101 × 102 × 103 × 104.
(17.) 23 × 231 × 306 × 462.
(18.) 93 × 361 × 792 × 2856.

Ex. 9.

(1.) Multiply thirty-eight thousand two hundred and seventy-four by three hundred and sixty-seven.

(2.) Multiply eight millions eight hundred and seventy-seven thousand six hundred and sixty-five by one hundred and ninety-eight thousand and seventy.

(3.) *(3005 + 1865)* × (3005—1865); (49 × 57)—(36 × 28).

(13.) Multiply [illegible]
(14.) „ [illegible]
(15.) „ [illegible]
(16.) „ [illegible]
(17.) „ [illegible]
(18.) „ [illegible]
(19.) „ [illegible]
(20.) „ [illegible]
(21.) [illegible] × [illegible]
(22.) [illegible] × 7 ×
(23.) [illegible] × 7 ×
(24.) [illegible] × 9 ×

2. *When the*

Rule.—Set the mu
under units.

Then, beginning at th
the multiplicand by the

Next, multiply by th
hand figure in the tens'
multiply by.

Proceed in the same w
bering to put the right
the figure multiplied by.

Lastly, add all the pro
product.

Example.—Multiply 6[illegible]

[illegible]

s to supply London with butter. How many pounds of nually consumed in that city?

azier bought 89 oxen at £13 each, and sold 37 of them at nd the rest at £12 each. What did he gain by his bargain?

at Britain and Ireland contain about 122,550 square miles l the population is reckoned at 237 persons to every square l the whole population of the two islands?

oy can point 13,579 pins in an hour. How many can he y-two weeks, each six days of nine hours?

ere are 176 houses in a street in London, 98 of which con- average 9 persons each, and the rest 14 persons. How many street?

cistern can be emptied by two pipes, one running 14 gallons er 15 gallons a minute. They were both opened together minutes, and then the cistern, which had been quite full, d? How many gallons did it hold when full?

V. DIVISION.

ION is the method of finding how often one number ned in another.

umber to be divided is called the Dividend: the **by which we divide is called the Divisor**: and the **answer is called the Quotient.**

n of Division is ÷ (di[illegible] ld shows that the number **of the sign is to be** [illegible] e one to the right; thus. **4, that is, twelve** [illegible] *equals* four.

es the Divide[illegible] e Divisor; thus.

$\frac{84}{12}$ [illegible] = **Quotient.**

Wh[illegible] *not greater than* 12.

[illegible] o the left of the dividend; then he divisor is conta[illegible] the gures of the divide[illegible] he

(4) How many oranges in 113 boxes, each containing 179 oranges?

(5.) How many words in an index of 79 pages, each page containing 5 columns of 148 words?

(6.) How many passengers in 82 trains of 7 carriages, each carriage holding 29 passengers?

(7.) How many words in a book of 792 pages, each page containing on an average 69 lines and each line 13 words?

(8.) How many persons can be seated in a church having 97 pews holding 8 persons each, 56 holding 6 each, 38 holding 4 each, besides 119 free seats?

(9.) A merchant begins business with £3500, and gains on an average £879 per annum, what will he have at the end of 19 years?

(10.) There are 27 regiments in an army, each regiment consists of 8 companies of 107 men: how many men are there in the whole army?

(11.) If you travel by railway for 17 hours at 38 miles per hour; then go by steam-boat 59 hours at 13 miles per hour; then by coach 11 hours at 8 miles per hour, and lastly walk 4 hours at 3 miles per hour: how far have you travelled?

(12.) Find the difference between the square of 999 and the cube of 99?

(13.) The Earth is distant from the Sun 91,841,100 miles, and the planet Neptune is nineteen times farther off. At what distance from the Sun does Neptune revolve?

(14) Ten thousand bricks are to be carted. When 14 carts have carried off 288 each, and 13 others 420 each, how many remain to be carted?

(15.) In a large building there were 67 windows; each window had 12 rows of panes, and each row contained 8 panes. How many panes of glass were there in the whole?

(16.) A railway company has 75 engines, 547 carriages, and 819 trucks. Each engine has eight wheels, each carriage six, and each truck four. On the average, there are fifteen spokes to each wheel. How many spokes are there altogether?

(17.) Find the difference between the square of one thousand and nine, and the cube of one hundred and nine.

(18.) A cattle dealer went to a fair, with £600 to lay out. He bought 28 oxen at £17 each, and 17 hogs at £6 each. How many pounds did he carry home of his money?

(19.) A good cow yields 167 pounds of butter *per annum*. It takes

225,000 cows to supply London with butter. How many pounds of butter are annually consumed in that city?

(20.) A grazier bought 89 oxen at £13 each, and sold 57 of them at £17 each, and the rest at £12 each. What did he gain by his bargain?

(21.) Great Britain and Ireland contain about 122,550 square miles of land, and the population is reckoned at 237 persons to every square mile. Find the whole population of the two islands?

(22.) A boy can point 13,579 pins in an hour. How many can he point in fifty-two weeks, each six days of nine hours?

(23.) There are 176 houses in a street in London, 98 of which contain on an average 9 persons each, and the rest 14 persons. How many live in the street?

(24.) A cistern can be emptied by two pipes, one running 14 gallons, and the other 15 gallons a minute. They were both opened together for fifteen minutes, and then the cistern, which had been quite full, was emptied? How many gallons did it hold when full?

V. DIVISION.

Division is the method of finding how often one number is contained in another.

The number to be divided is called the **Dividend**; the number by which we divide is called the **Divisor**; and the result or answer is called the **Quotient**.

The sign of Division is ÷ (divided by) and shows that the number to the left of the sign is to be divided by the one to the right; thus, $12 \div 3 = 4$, that is, twelve *divided by* three *equals* four.

Sometimes the Dividend is placed over the Divisor; thus,

$$\frac{84}{12} = 7; \text{ or } \frac{\textbf{Dividend,}}{\textbf{Divisor.}} = \textbf{Quotient.}$$

1. *When the divisor is not greater than* 12.

Rule.—Place the divisor to the left of the dividend; then consider how many times the divisor is contained in the first or first and second figures of the dividend, and set the number underneath. If any number remains, carry it, as so many tens, to the next figure of the dividend and divide as before. Place each figure of the quotient under that of the same *local value* in the dividend.

If the divisor is not contained in the dividend *exactly*, the remainder may be set down over the divisor in the form of a fraction.

Example.—Divide 378697 by 7.

$$\begin{array}{r} 7\,)\underline{378697} \\ 54099\,\frac{4}{7} \end{array}$$

Worked thus :—*Seven* in *three* (hundred thousands) cannot be found; then *seven* in *thirty-seven* (ten thousands) *five* (ten thousands) and *two* (ten thousands) *remaining.* Put down 5 (ten thousands) and carry 2 (ten thousands).

Seven in *twenty-eight* (thousands) *four* (thousands). Set down 4 (thousands).

Seven in *six* (hundreds) cannot be found. Put down 0 and carry 6 (hundreds).

Seven in *sixty-nine* (tens) *nine* (tens) and *six* (tens) remaining. Put down 9 (tens) and carry six (tens).

Seven in *sixty seven* (units) *nine* (units) and *four* (units) remaining. Put down 5 (units) and place the remainder 4 over the divisor as $\frac{4}{7}$, that is four-sevenths of a unit.

Proof.—Multiply the quotient by the divisor and add the remainder, if any; and the result will be the same as the dividend, if the working is correct.

Ex. 10.

(1.) 3)2138754
(2.) 4)2899276
(3.) 5)1759062
(4.) 5)21318760
(5.) 6)34190592
(6.) 7)62464332
(7.) 7)34464969
(8.) 8)53422712
(9.) 9)33878115
(10.) 11)54159237
(11.) 7)52778922
(12.) 12)95210580

(13.) Divide 373248 separately by 2, 3, 4, 5, 6.
(14.) „ *2460375* „ 2, 3, 5, 7, 8.

(15.) Divide 6842753980 separately by 2, 4, 6, 8, 12.
(16.) " 7839150462 " 3, 5, 7, 9, 11.
(17.) " 8965432710 " 5, 7, 8, 9, 11.
(18.) " 7930865430 " 3, 5, 7, 9, 11.

(19.) Divide 6187459680 successively by 2, 3, 4, 5, 6.
(20.) " 45995995584 " 7, 8, 9, 11, 12.
(21.) " 21258732 " 11, 11, 12, 3.
(22.) " 2985984 " 12, 12, 2, 8.
(23.) " 1627531080 " 3, 7, 8, 12.
(24.) " 874625472 " 7, 8, 9, 12.

2. *When the divisor is greater than* 12.

Rule.—If the divisor is a **Composite number** made up of factors none of which are greater than twelve, divide the dividend by the several factors *successively*, as in the last exercise.

Should there be several remainders, the *true* remainder will be obtained by multiplying the last remainder by the preceding divisor and adding to the product the preceding remainder.

This process has been termed **Double or Composite Division.**

A Composite number is a number formed by the successive multiplication of two or more numbers, which are called its **Factors**, thus 105 is a composite number, and its factors are 3, 5, 7, for $3 \times 5 \times 7 = 105$.

Example.—Divide 9178964 by 27 and 105

$$\begin{array}{l} 3\,\underline{)9178964} \\ \left.\begin{array}{r} 9\,\underline{)3059654} \div 2 \\ 339961 \div 5 \end{array}\right\} 17 \end{array}$$

$(5 \times 3) + 2 = 17.$

$$\begin{array}{l} 3\,\underline{)9176964} \\ \left.\begin{array}{l} 5\,\underline{)3059654} \div 2 \\ \left.\begin{array}{r} 7\,\underline{)\ 611930} \div 4 \\ 87418 \div 4 \end{array}\right\} 24 \end{array}\right\} 74 \end{array}$$

$(4 \times 5) + 4 = 24.$

$(24 \times 3) \div 2 = 74$

Ex. 11.

(1.) 6235792834 ÷ 14, 32, 36. (2.) 8234567923 ÷ 15, 16, 18.
(3.) 7906284793 ÷ 24, 25, 27. (4.) 4837162849 ÷ 44, 45, 48.

(5.) 4103817128 $\div$ 54, 56, 63. (6.) 7923457923 $\div$ 64, 66, 72.

(7.) 9324063792 $\div$ 72, 84, 96. (8.) 8243579234 $\div$ 108, 132, 144.

(9.) 7843155120 $\div$ 24, 72, 168. (10.) 2593245675 $\div$ 25, 75, 125.

(11.) 3246789409 $\div$ 44, 176, 264. (12.) 8567469300 $\div$ 63, 126, 315.

(13.) 942954496 $\div$ 96, 288, 384. (14.) 6576825312 $\div$ 144, 432, 576.

(15.) 8027525844 $\div$ 108, 432, 756. (16.) 9673245698 $\div$ 121, 484, 968.

(17.) 8943679234 $\div$ 132, 264, 396. (18.) 7936256837 $\div$ 147, 189, 175.

(19.) 7934256937 $\div$ 168, 225, 336. (20.) 5007006419 $\div$ 112, 448, 896.

(21.) 8293476238 $\div$ 315, 324, 704 ($= 8 \times 8 \times 11$), 864 ($= 8 \times 9 \times 12$).

(22.) 9765432897 $\div$ 343, 4096, 1089, 3267 ($= 3 \times 9 \times 11 \times 11$).

If the divisor be a **Prime number** (*i.e.* one that has no factors), or one the factors of which are greater than 12, the method called **Long Division** must be used.

RULE.—Place the divisor to the left of the dividend as before. Take the least number of figures on the left of the dividend that will contain the divisor, and find how many times they contain it; place the number so found on the right of the dividend as the first figure of the quotient.

Multiply the divisor by that figure and set down the product under the figures taken from the dividend, and subtract it from them.

To the right of the remainder set the next figure of the dividend, and divide the number thus formed by the divisor and proceed as before, and so on till all the numbers of the dividend have been used.

EXAMPLE.— Divide 96875 by 57.

```
57)96875(1699 32/57
   57
   ---
   398                Proof    1699
   342                           57
   ---                        -----
    567                       11893
    513                       8495
    ---                          32
     545                      -----
     513                      96875
     ---                      -----
      32
```

Worked thus :—*Ninety-six* (thousands) divided by *fifty-seven* gives *one* (thousand) ; put the 1 (thousand) in the quotient.

Fifty-seven multiplied by *one* (thousand) equal *fifty-seven* (thousands). Set down 57 under the 96.

Take *fifty-seven* (thousands) from *ninety-six* (thousands) there remain *thirty-nine* (thousands).

Bring down 8 (hundreds).

Then 398 (hundreds) divided by 57 give 6 (hundreds). Put the 6 (hundreds) in the quotient.

Fifty-seven multiplied by *six* (hundreds) equals *three hundred and forty-two* (hundreds). Set down 342 under 398.

Take 342 (hundreds) from 398 (hundreds) there remain 56 (hundreds).

Bring down 7 (tens).

Then 567 (tens) divided by 57 equals 9 (tens). Put the 9 (tens) in the quotient.

Fifty-seven multiplied by *nine* (tens) equals *five hundred and thirteen* (tens). Set down 513 under 567.

Take 513 (tens) from 567 (tens) there remain 54 (tens).

Bring down 5 (units).

Then 545 (units) divided by 57 give 9 (units). Put the 9 (units) in the quotient.

Fifty-seven multiplied by *nine* (units) equals *five hundred and thirteen* (units). Set down 513 under 545.

Take 513 (units) from 545 (units) there remain 32.

32 (units) divided by 57 (units) equals $\frac{32}{57}$ (of one unit).

The preceding operation would stand thus, if the figures were written down with their *local* values:

$$
\begin{array}{r}
57)96875(1000 + 600 + 90 + 9 \\
\underline{57000} \\
39875 \\
\underline{34200} \\
5675 \\
\underline{5130} \\
545 \\
\underline{513} \\
32
\end{array}
\qquad
\begin{array}{lrcr}
\text{Proof} & 1000 \times 57 & = & 57000 \\
 & 600 \times 57 & = & 34200 \\
 & 90 \times 57 & = & 5130 \\
 & 9 \times 57 & = & 513 \\
 & \text{remainder} & = & \underline{32} \\
 & & & 96875
\end{array}
$$

Proof.—Multiply the quotient by the divisor and add the remainder, the result will be the same as the dividend

Ex. 12.

(1.) 3765897 ÷ 17, 19, 23.
(2.) 4613578 ÷ 31, 37, 39.
(3.) 5123495 ÷ 41, 43, 46.
(4.) 3954371 ÷ 47, 51, 59.
(5.) 7923406573 ÷ 62, 65, 67.
(6.) 7834562935 ÷ 69, 83, 87.
(7.) 7924563894 ÷ 71, 73, 79.
(8.) 8246735849 ÷ 92, 93, 97.
(9.) 5502669904 ÷ 647, 809.
(10.) 2070784080 ÷ 305, 915.
(11.) 5185854696 ÷ 8888, 9616.
(12.) 716117166 ÷ 999, 7918.
(13.) 4823604456 ÷ 4884, 5861.
(14.) 3417870411 ÷ 6393, 9983.
(15.) 3456793728 ÷ 4536, 48896.
(16.) 4637064288 ÷ 8991, 57606.
(17.) 3791977069738338 ÷ 475869.
(18.) 3331822334991980 ÷ 859476.
(19.) 1414103697906795 ÷ 467985.
(20.) 985903616419668 -÷ 589476.

If the divisor ends in one or more ciphers, the division may, *for convenience*, be arranged as below; the ciphers being marked off from the divisor, and a corresponding number of figures from the dividend, the rest of the operation is performed as before.

Example.—Divide 896573100 by 30, 630, 5730.

```
                                           573,0)89657310,0(156470
                                                 573
     3,0)89657310,0                              ----
         ----------                              3235
          29885770                               2865
         ----------                              ----
                                                  3707
      ⎧ 9,0)89657310,0                            3438
 630  ⎨      ----------                           ----
      ⎩   7)9961923 + 3 ⎫ 3,0     1                2693
            ----------  ⎬ ----- = --               2292
            1425989 + 0 ⎭ 63 0    21               ----
                                                    4011
                                                    4011
                                                    ----
                                                       0
```

Ex. 13.

(1.) 74357680 ÷ 40, 60, 80.
(2.) 64831410 ÷ 30, 70, 90.
(3.) 731460840 ÷ 120, 450.
(4.) 746381050 ÷ 370, 1480.
(5.) 722299000 ÷ 1080, 23000.
(6.) 793245620 ÷ 2700, 39000.
(7.) 523480000 ÷ 9200, 33900.
(8.) 689725400 ÷ 6010, 76900.
(9.) 16175870400 ÷ 57900.
(10.) 24504402000 ÷ 78000.
(11.) 321004568646 ÷ 64200.
(12.) 2706578471 ÷ 54330.

Ex. 14.

(1.) A plantation containing 21,312 trees, consisted of 192 rows. How many trees were in each row?

(2.) A person's income is £1568 per annum. What is that per week?

(3.) Divide £64827 equally between 27 persons.

(4.) If a boy can point 768,000 pins in six days of eight hours each, how many can he point in an hour?

(5.) Six boys have respectively 72, 74, 76, 78, 80, and 82 marbles. Find the average share.

(6.) Six towns have respectively the following populations:—3155, 8671, 4671, 7632, 2169, and 2864. Find the average population?

(7.) Seven parishes paid respectively the following poor's rates:—£689, £567, £283, £364, £435, £742, and £693. What was the average payment?

(8.) Divide 2000 nuts equally among 86 children. How many will there be remaining?

(9.) A gentleman gave away 432 shillings among 12 men and 18 women, giving to each man three times as much as to each woman. How many shillings did he give to each?

(10.) Out of a regiment consisting of 1344 men, 64 were officers. How many privates were there to each officer.

(11.) Scotland contains 33 counties and 3,061,117 inhabitants. Find the average population of a county?

(12.) The produce of 110 sugar canes is 6 pounds of sugar. How many pounds were raised from a plantation in which there were 1,233,210 canes?

(13.) For one page of a book containing 36 lines, 2052 letters were used. How many letters does that average per line?

(14.) The length of a railway is 81 miles, and the cost was a million of money all but £55. What did it cost per mile?

(15.) The dividend of a division sum is 734097143, the quotient 127824, and the remainder 3911. What is the divisor?

(16.) The product of a multiplication sum is 8055504, and the multiplicand is 3496. What is the multiplier?

(17.) $\frac{584 + 273 - 17}{28}$; $\frac{5086 + 548 - 37}{478 - 285}$; $\frac{3376 - 713 + 6606}{768 - 1090 + 345}$.

(18.) $(347 + 515) \div (8818 - 8742)$; $(1001 - 987) \times (177 \div 42)$.

(19) $\frac{(4 \times 194) + (9 \times 54) - (97 + 51)}{412 - 286}$; $\frac{812 \times (764 - 388) - 1008}{(60 - 132 + 116) \times 28}$.

(20.) $\frac{1286 - 68 + 457 - 89 - 275}{576 - 189 + 23 - 287} \times \frac{576 - 1001 + 426}{839 + 470 - 1308}$

(21.) In how many days will a traveller driving at the rate of six miles an hour, for eight hours a day, complete a journey of 500 miles?

(22.) From the Norman conquest (A.D. 1066) to the death of William IV. (A.D. 1837), thirty-four sovereigns reigned over England. What was the average length of a reign?

(23.) A farmer, who owned 1500 sheep, had his number reduced to 1227 in three weeks. What was his average daily loss?

(24.) The circumference of the earth at the equator is about 24,904 miles. How many hours would it take a locomotive, travelling 28 miles per hour, to complete the circuit?

(25.) An omnibus conductor has in one day 66 passengers paying 3d. each, 97 paying 4d. each, and 33 paying 6d. each. How many pence does he receive in all, and what is the average fare of each?

ARITHMETICAL TABLES.

MONEY.

I.—CURRENT COINAGE.

4 Farthings (*f.*)	make	1 Penny	1*d.*
12 Pence	—	1 Shilling	1*s.*
20 Shillings	—	1 Sovereign or Pound	£1

One farthing, or *fourth* of a penny, is denoted by ¼*d.*; *two* farthings, or a *halfpenny*, by ½*d.*; *three* farthings, by ¾*d.*

A *Florin* = 2 Shillings.
A *Crown* = 5 Shillings.
A *Guinea* = 21 Shillings.

Pence Table.

d.		*s.*	*d.*	*d.*		*s.*	*d.*	*d.*		*s.*	*d.*
12	make	1	0	50	make	4	2	100	make	8	4
18	—	1	6	60	—	5	0	108	—	9	0
20	—	1	8	70	—	5	10	110	—	9	2
24	—	2	0	72	—	6	0	120	—	10	0
30	—	2	6	80	—	6	8	130	—	10	10
36	—	3	0	84	—	7	0	132	—	11	0
40	—	3	4	90	—	7	6	140	—	11	8
48	—	4	0	96	—	8	0	144	—	12	0

Shilling Table.

s.		£	*s.*	*s.*		£	*s.*	*s.*		£	*s.*
20	make	1	0	60	make	3	0	100	make	5	0
30	—	1	10	70	—	3	10	200	—	10	0
40	—	2	0	80	—	4	0	500	—	25	0
50	—	2	10	90	—	4	10	1000	—	50	0

WEIGHT OF COINS.

GOLD.

Sovereign.........	123¼ Grains.	Half Sovereign	61½ Grains.

SILVER.

Crown	436	Grains	Sixpence...............	44	Grains.
Half Crown	218	„	Fourpence	29	„
Florin	174½	„	Threepence............	22	„
Shilling	87¼	„			

BRONZE.

48 pennies are coined out of one pound Avoirdupois.

The sovereign constitutes the standard of value in Great Britain, its weight being 123·25 grains, consisting of 113 1 grains of pure gold and 10 15 grains of copper alloy.

The gold coins of Great Britain are made from a metal of which 22 parts in 24 are pure gold, and 2 parts alloy, a mixture of copper and silver, added for the purpose of giving greater hardness and durability.

A pound Troy of this metal is coined into $46\frac{29}{40}$ sovereigns = £46 14s. 6d.; the Mint price of standard gold is therefore one-twelfth of that sum per ounce, *viz.* £3 17s. 10½d., and as there are 11 oz. of pure gold in 12 oz. of the standard, the value per ounce of pure gold is $\frac{1}{11}$ of £46 14s. 6d = £4 4s. $11\frac{5}{11}$d., or about £4 5s.

The standard silver coin is made of a metal of which 37 parts in 40 are pure silver, and 3 parts copper. 66s. are coined from a pound Troy of this metal, so that the Mint price per ounce of standard silver is 5s. 6d., which is about 10 per cent. above its real value; the market price of mint silver being about 5s. per ounce.

The bronze metal used for coins consists of 95 parts copper, 4 tin, and 1 zinc, in every 100. Bronze coins are not a legal tender for more than 12d., nor silver for more than 40s., the gold coinage being the standard coin of the realm.

OBSOLETE ENGLISH COINS.

A Moidore	= 27s.	A Guinea	= 21s. 0d.	An Angel	= 10s. 0d.
A Jacobus	= 25s.	A Pistole	= 16s. 0d.	A Half Guinea	= 10s. 6d.
A Carolus	= 23s.	A Mark	= 13s. 4d.	A Noble	= 6s. 1d.

II.—DECIMAL COINAGE.

10 Mils, (*m.*) = 1 Cent (*c.*)
10 Cents = 1 Florin (*fl.*)
10 Florins = 1 Pound (£)

TABLE OF SHILLINGS, PENCE, AND FARTHINGS, REDUCED TO MILS, *Or, Thousandths of a Pound.*

Sum.		*Mils.*	*Sum.*		*Mils.*	*Sum.*		*Mils.*
0¼d.	=	·001	0s. 10d.	=	·042	11s.	=	·550
0½	=	·002	0 11	=	·046	12	=	·600
0¾	=	·003	1 0	=	·050	13	=	·650
1	=	·004	2 0	=	·100	14	=	·700
2	=	·008	3 0	=	·150	15	=	·750
3	=	·013	4 0	=	·200	16	=	·800
4	=	·017	5 0	=	·250	17	=	·850
5	=	·021	6 0	=	·300	18	=	·900
6	=	·025	7 0	=	·350	19	=	·950
7	=	·029	8 0	=	·400	20	=	1·000
8	=	·033	9 0	=	·450			
9	=	·037	10 0	=	·500			

The foregoing table may be used as follows:—

EXAMPLE.—Express 17s. 5½d. in mils.

Thus, 17s. 0d. = ·850
0 5 = ·021
0 0½ = ·002

17 5½ = ·873 = 8 fl. 7 c. 3 m.

WEIGHT.

I.—AVOIRDUPOIS WEIGHT.

Used for all articles except gold, silver, and jewels.

16 Drams (*dr.*) make	1 Ounce	1 *oz.*
16 Ounces	1 Pound	1 *lb.*
14 Pounds	1 Stone	1 *st.*
28 Pounds	1 Quarter	1 *qr.*
4 Quarters	1 Hundredweight	1 *cwt.*
20 Hundredweight	1 Ton	1 *ton.*

1 lb. Avoirdupois = 7000 Grains.
1 oz. „ = 437½ „

II.—TROY WEIGHT.

Used for weighing Gold and Silver, and in Philosophical Experiments.

24 Grains (*gr.*) make	1 Pennyweight	1 *dwt.*
20 Pennyweights	1 Ounce	1 *oz.*
12 Ounces	1 Pound	1 *lb.*

1 lb. Troy = 5760 Grains.
1 oz. „ = 480 „

III.—APOTHECARIES WEIGHT.

Used only in mixing medicines.

20 Grains (*gr.*) make	1 Scruple	1 *sc.* or ℈.
3 Scruples	1 Dram	1 *dr.* or ʒ.
8 Drams	1 Ounce	1 *oz.* or ℥.
12 Ounces	1 Pound	1 *lb.*

In the above table, the *grain, ounce,* and *pound* are the same as in Troy Weight; but in the system of weights adopted in 1862 by the General Medical Council, the *scruple* and *dram* have been omitted, and the *ounce* is the *Avoirdupois* ounce of 437½ grains.

IV.—MISCELLANEOUS MEASURES OF WEIGHT.

1 Stone (Butcher's meat) = 8 lbs.
1 Truss of Straw .... = 36 lbs.
1 Truss of Hay .... = 56 lbs.
1 Load .... = 36 trusses.

1 Sack of Coals ... = 2 cwt.
1 Fother of Lead... = 19½ cwt.
1 Pack of Wool ... = 240 lbs.
1 Firkin of Butter = 56 lbs.

The *Imperial* Standard of Weight is the weight of a cubic inch of distilled water, which, when the Barometer stands at 30in., and the Thermometer at 62°, is 252.458 grains.

LENGTH.

I.—LONG MEASURE.

12 Inches (*in.*) make	1 Foot	1 *ft.*
3 Feet	1 Yard	1 *yd.*
5½ Yards	1 Pole	1 *po.*
40 Poles, or 220 yards	1 Furlong	1 *fur.*
8 Furlongs, or 1760 yards	1 Mile	1 *mi.*
3 Miles	1 League	1 *lea.*

N.B.—In reducing yards to poles, it is best to bring the yards to half-yards, by multiplying by 2, and then to divide by 11, for 11 *half-yards* equal 1 pole. The remainder, if any, will be *half-yards.*

II.—CLOTH MEASURE.

2¼ Inches make	1 Nail	1 *nl.*
4 Nails	1 Quarter (of a yard)	1 *qr.*
3 Quarters	1 Flemish ell	1 *Fl. ell.*
4 Quarters	1 Yard	1 *yd.*
5 Quarters	1 English ell	1 *En. ell.*
6 Quarters	1 French ell	1 *Fr. ell.*

N.B.—In reducing inches to nails, multiply by 4, and divide by 9, for 9 quarters of an inch equal 1 nail. Whatever remains will be *quarters of an inch.*

III.—MISCELLANEOUS MEASURES OF LENGTH.

1 Palm ...... = 3 in.
1 Hand ...... = 4 in.
1 Span ...... = 9 in.
1 Scotch ell . = 37 in.
1 Pace......... = 5 ft.
1 Fathom ... = 6 ft.

1 Line ...... = $\frac{1}{12}$ in.
1 Link ...... = 7.92 in.
1 Chain ... = 100 links or 66 ft.
1 Mile ...... = 80 chains.
1 Degree at the Equator = 69 1 statute miles. / 60 geog. miles.

The *Imperial* Standard of Length is the length of a second's pendulum, which in the latitude of London, at the level of the sea, and a temperature of 62° Fahrenheit is 39.1393 inches.

CAPACITY.

I.—IMPERIAL MEASURE.

4 Gills	make	1 Pint	1 *pt.*
2 Pints	,,	1 Quart	1 *qt.*
4 Quarts	,,	1 Gallon	1 *gall.*
2 Gallons	,,	1 Peck	1 *pk.*
4 Pecks	,,	1 Bushel	1 *bu.*
8 Bushels	,,	1 Quarter	1 *qr.*
5 Quarters	,,	1 Load	1 *ld.*

The *Peck*, *Bushel*, *Quarter*, and *Load*, are used for dry goods only.

II.—OBSOLETE MEASURES OF CAPACITY.

1 Barrel of Beer = 36 gals.
1 Hogshead ...... = 54 gals.
1 Hogshead Wine = 63 gals.
1 Pipe or Butt ... = 2 hhds.
1 Tun.............. = 2 pipes.

1 Pottle .... = 2 quarts.
1 Strike .... = 2 bush.
1 Coomb .... = 4 bush.
1 Sack .... = 3 bush.
1 Chaldron ...... = 12 sacks.

The Imperial measure, by act of parliament, superseded all other measures of capacity.

The Standard Imperial gallon contains 10 pounds avoirdupois of distilled water at the temperature of 62° Fahrenheit, which exactly fills a space of 277.274 cubic inches.

TIME.

TIME MEASURE.

60 Seconds (*sec.*) make	1 Minute	1 *min.*
60 Minutes	1 Hour	1 *hr.*
24 Hours	1 Day	1 *dy.*
7 Days	1 Week	1 *wk.*
4 Weeks	1 Month	1 *mo.*
12 Months 52 Weeks 1 day or 365 Days	1 Common Year	1 *co. yr.*
365¼ Days	1 Julian Year	1 *Ju. yr.*
366 Days	1 Leap Year	1 *lp. yr.*

The year is also divided into 12 months, called *Calendar* Months, which contain unequal numbers of days :—

January......	31	May.........	31	September...	30
February ...	28	June.........	30	October......	31
March	31	July.........	31	November...	30
April.........	30	August......	31	December ...	31

In leap year, February has 29 days.

A *lunar* month or period from new moon to new moon is 29 days 12 hours 44 minutes 2.87 seconds.

A *solar* year or period from vernal equinox to vernal equinox is 365.24224 days = 365 days 5 hours 48 minutes 49.7 seconds.

The difference between a *Julian* and *Solar* year = .00776 day, this will amount in 400 years to 3.104 days; that is 400 Julian years will be 3.104 days longer than 400 true Solar years. To allow for this it has been arranged, that when the number of a year denoting a complete century is *not* divisible by 400, that year shall *not* be reckoned a leap year; thus 3 days will be dropped in every 400 years, leaving only an error of .104 day in that time, or about a day in 4000 years.

SURFACE, SOLIDITY, INCLINATION.

I.—SQUARE MEASURE.

144 Square Inches make	1 Square Foot	1 *sq. ft.*
9 Square Feet	1 Square Yard	1 *sq. yd.*
30¼ Square Yards	1 Square Rod, Pole, or Perch	1 *P.*
40 Perches	1 Rood	1 *R.*
4 Roods (4840 sq. yds.)	1 Acre	1 *A.*
640 Acres..................	1 Square Mile	1 *sq. ml.*

484 Square Poles ...... } make 1 Square Chain.
10000 Square Links ...... }
10 Square Chains ...... 1 Acre.

All surfaces are measured in square inches, square feet, &c. The square inch is a square whose sides are each an inch in length; a square foot is a square whose sides are each a foot in length; and so on.

N.B.—To divide by 30¼ multiply by 4 and divide by 121, for 121 quarters = 30¼.

II.—CUBIC MEASURE.

1728 Cubic Inches make	1 Cubic Foot	1 *cub. ft.*
27 Cubic Feet	1 Cubic Yard	1 *cub. yd.*

1 Ton (in shipping) = 42 Cubic Feet.
1 Ton or Load of Rough Timber = 40 ,, ,,
1 Ton or Load of Hewn Timber = 50 ,, ,,

The contents of all solids are measured in cubic inches, cubic yards, &c.

A cubic inch is a cube or six-sided solid, each of whose edges is an inch in length.

III.—ANGULAR MEASURE.

60 Seconds (″) make	1 Minute	1′
60 Minutes	1 Degree	1°
360 Degrees	1 Circle.	

A degree of latitude or of any *great* circle on the earth is equal to 69$\frac{1}{10}$ English miles.

DECIMAL OR METRIC SYSTEM OF WEIGHT AND MEASURE.

An Act of Parliament passed in 1864, legalises the use of the Metric System of Weights and Measures in this country. This system, which was established in France in 1799, takes its name from the **Mètre** or unit of length, which is equal to 39.37079 English inches. This Mètre is the ten-millionth part of the distance from the equator to the pole, and from it all the other measures are derived.

The **Mètre**, the unit of *length*, is nearly 1$\frac{1}{10}$ yards.

The **Are**, the unit of *surface*, is a square décamètre, or nearly 120 square yards.

The **Litre**, the unit of *capacity*, is a cubic décimètre, or about 1$\frac{3}{4}$ pints, or 61,0271 cubic inches.

The **Stère**, the unit of *solidity*, is a cubic mètre, or about 35$\frac{1}{2}$ cubic feet.

The **Gramme**, the unit of *weight*, is the weight of a cubic centimètre of distilled water, at 32° Fahr., or about 15$\frac{1}{2}$ grains.

The larger measures are obtained by multiplying the unit by 10, 100, 1000, 10,000, and are denoted by Greek prefixes ;—*Myria-*, 10,000 ; *Kilo-*, 1000 ; *Hecto-*, 100 ; *Deka-*, 10.

The smaller measures are obtained by dividing the unit by 10, 100, 1000, and are denoted by Latin prefixes :—*Deci-*, $\frac{1}{10}$; *Centi-*, $\frac{1}{100}$; *Milli-*, $\frac{1}{1000}$.

The following tables shew at one view the arrangement of the Metric System and the English equivalents of the several measures.

METRIC SYSTEM.

	Length.	*Surface.*	*Solidity.*	*Capacity.*	*Weight.*
Multiples.					
Myria-	10000	...	...	...	10000
Kilo-	1000	...	...	1000	1000
Hecto-	100	100	...	100	100
Deca-	10	...	10	10	10
UNITS	**Mètre.**	**Are.**	**Stère.**	**Litre.**	**Gramme.**
Divisions.					
Deci-	$\frac{1}{10}$	...	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
Centi-	$\frac{1}{100}$	$\frac{1}{100}$	...	$\frac{1}{100}$	$\frac{1}{100}$
Milli-	$\frac{1}{1000}$	...	...		$\frac{1}{1000}$

LENGTH.

Millimètre .... = ·001 Mètres = ·03937079 inches.
Centimètre .... = ·01 „ = ·3937079 „
Decimètre .... = ·1 „ = 3·937079 „
METRE .... = 1. „ = 39·37079 „
Decamètre .... = 10 „ = 10·936331 yards.
Hectomètre ...... = 100 „ = 109·36331 „
Kilomètre .... = 1000 „ = 1093·6331 „
Myriamètre ...... = 10000 „ = 10936·331 „

Long distances are usually reckoned in Kilomètres.

1 Kilomètre = 1100 yards, or five-eighths of an English mile, nearly.

1 Inch = 2·5399 Centimètres.
1 Foot = 3·0479 Decimètres.

1 Yard = ·9143 Mètre.
1 Mile = 1·6093 Kilomètre.

SURFACE.

Centiare = ·01 Are = 1·1960333 Square Yards.
ARE ... = 1. „ = 119·603326 „ „
Hectare = 100 „ = 11960·332602 „ „

Large surfaces are usually measured by Hectares.

1 Hectare = $2\frac{1}{2}$ acres, nearly.
1 Centiare = 1 square mètre.

1 Square Yard = ·8361 Square Mètres.
1 „ Pole = 25·2920 „ „

1 Rood = 10·1168 Ares.
1 Acre = ·4046 Hectares.

SOLIDITY.

Decistère = .1 STÈRE = 3·5316581 cubic feet.
STÈRE ... = 1 „ = 35·3165807 „ „
Decastère = 10 „ = 353·1658074 „ „

The **Stère** is used chiefly in measuring wood for fuel.

1 Cubic Foot = 28·3153119 Cubic Decimètres.
1 „ Inch = 16·3861759 „ Centimètres.

CAPACITY.

Centilitre = ·01 Litre = ·017608 pints.
Decilitre = ·1 „ = ·176077 „
LITRE = 1. „ = 1·760773 „
Decalitre = 10 „ = 2·200966 gals.
Hectolitre = 100 „ = 22·009667 „
Kilolitre = 1000 „ = 220·096676 „

Large quantities are often measured by Hectolitres.
1 **Hectolitre** is about $2\frac{3}{4}$ bushels.

1 Quart = 1·1358 Litres.
1 Gallon = 4·5434 „
1 Bushel = 36·3476 Litres.
1 Quarter = 2·9078 Hectolit.

WEIGHT.

Milligramme = ·001 Gramme = ·015432 grains.
Centigramme = ·01 „ = ·154323 „
Decigramme = ·1 „ = 1·543235 „
GRAMME = 1. „ = 15·432349 „
Decagramme = 10 „ = 154·323488 „
Hectogramme = 100 „ = ·220462 lbs.
Kilogramme = 1000 „ = 2·204621 „
Myriagramme = 10000 „ = 22·046213 „

Large quantities are generally reckoned in Kilogrammes.
1 **Kilogramme** is about $2\frac{1}{4}$ lbs.
1 **Quintal** or 100 Kilogrammes = 2 cwt. nearly.
1 **Millier** or 1000 „ = 1 ton.

1 Grain = ·0648 Gramme.
1 Oz. Troy = 31·1035 „
1 lb. Avoird. = ·4536 Kilog.
1 cwt. „ = 50·8024 „

VI.—REDUCTION.

Reduction is the method of changing a **Concrete** number from one denomination to another without altering its value.

An **abstract number** refers to number in general without reference to any particular objects, as 4, 12, 20.

A **concrete number** denotes a number of particular objects, as 4 farthings, 12 pence, 20 shillings, 30 hundred-weight, &c.

Rule.—To reduce from a higher to a lower denomination,—**Multiply**.

To reduce from a lower to a higher denomination,—**Divide**.

The multiplier or divisor will always be that number of the lower denomination which is equal to one of the higher.

Example.—Reduce £5 14s. $3\frac{1}{2}$d. to farthings; and 789643 farthings to pounds.

(A)

£ *s.* *d.*
5 14 $3\frac{1}{2}$
20
114 shillings
12
1371 pence
4
5486 farthings.

(B)

Farthings.
4)789643
12)197410 $\frac{3}{4}$
2,0)1645,0 $10\frac{3}{4}$d.
£822 10s. $10\frac{3}{4}$.

(A) *Worked thus:*—Multiply the £5 by 20 (because 20s. = £1) and add the 14s.; this will come to 114s.

Multiply 114s. by 12 (because 12d. = 1s) and add the 3d.; this will come to 1371d.

Multiply 1371d. by 4 (because 4 farthings = 1d.) and add the 2 farthings or one half-penny; this will come to 5486 farthings.

(B.) *Worked thus:*—Divide 789643 farthings by 4 (because 4 farthings = 1d.); this will give 197410 pence and 3 farthings remaining.

Divide 197410 pence by 12 (because 12d. = 1s.); this will give 16450 shillings and 10 pence remaining.

Divide 16450 shillings by 20 (because 20s. = £1); this will give 822 pounds and 10 shillings remaining.

Ex. 15.

Money.

the following sums of money to farthings:—

£	s.	d.		£	s.	d.		£	s.	d.
144	2	9¾	(2.)	770	7	10½	(3.)	784	12	8¼
018	7	1¾	(5.)	720	1	1½	(6.)	700	7	10½
172	16	0	(8.)	866	16	9¾	(9.)	1122	2	6½
74	19	7¼	(11.)	2546	11	0¼	(12.)	1090	17	5¾

ow many half-pence in £86 13s. 4½d.; in £99 19s. 11½d.?
ow many half-pence in 4321 crowns; in 1170 half-crowns?
ow many three-penny pieces in 3749 florins; in £392 14s.

ow many farthings in 769 guineas; in 1461 guineas?
educe to half-crowns £29 7s. 6d.; £475 12s. 6d.; £7641

£36 15s. 0d. how many crowns and three-penny pieces?
78469 guineas how many four-penny pieces?
8376 half-sovereigns how many half-pence?

ny pounds in

	7680 s.	(2.)	98565 d.	(3.)	39469 f.
	6727 f.	(5.)	327970 d.	(6.)	1872640 d.
	0272 crowns.	(8.)	20541 d.	(9.)	80364 d.
	9567 groats.	(11.)	2129 hf.-crs.	(12.)	9765423 f.

	68 groats to threepences.	(14.)	1295 florins to hf.-crs.
	733 to guineas.	(16.)	38765 guineas to crs.
	150 groats to hf.-crs.	(18.)	76435 guineas to £.
	340 threepences to £.	(20.)	897624 d. to guineas.
	7392 f. to florins.	(22.)	13629 crowns to guineas.

a million farthings how many pounds?
seventy thousand and ninety half-pence how many pounds?
16960 crowns, as many half-crowns, and as many sixpences, pounds?

Ex. 16.

Weight.

avoirdupois)

	cwt. to lbs.	(2.)	56 tons to oz.
	cwt. 1 qr. to oz.	(4.)	1 qr. 11 lbs. 1 oz. to drs.
	cwt. 1 qr. 20 lbs to oz.	(6.)	15 cwt. 18 lbs. to lbs.

(7.) 27 tons 8 cwt. 3 qrs. 25 lbs. 7 oz. to ounces.
(8.) 27 cwt. 3 qrs. 4 lbs. 10 oz. 3 drs. to drams.
(9.) 115 tons 17 cwt. 2 qrs. 6 lbs. to lbs.
(10.) 11 tons 19 cwt. 0 qrs. 20 lbs. 8 oz. to drams.

(11.) 4193280 oz. to tons. (12.) 204144640 drs. to tons.
(13.) 111440 lbs. to tons. (14) 175344 oz. to cwt.
(15.) 1152912 drs. to cwt. (16.) 93184 drs. to qrs.
(17.) 1633632 oz. to cwt. (18.) 1111040 drs. to tons.

Reduce *(Troy)*

(1.) 17 lbs. 3 oz. 7 dwts. to grs. (2.) 28 lbs. 2 oz. 5 dwts. to dwts.
(3.) 77 oz. 19 dwts. 20 grs. to grs. (4.) 27 lbs. 3 dwts. 15 grs. to grs.
(5.) 41 lb. 8 oz. 11 dwts. to dwts. (6.) 20 oz. 16 dwts. 16 grs. to grs.
(7.) 51360 dwts. to lbs. (8.) 17496 grs. to lbs.
(9.) 28936 grs. to oz. (10.) 137760 grs. to oz.
(11.) 123000 grs. to lbs. (12.) 47965 dwts. to lbs.

(13.) 91 oz. 4 drs. 2 sc. to grs. (14.) 36 oz. 3 drs. 1 sc. to scs.
(15.) 27 drs. 2 sc. 17 grs. to grs. (16.) 30000 grs. to drams.
(17.) 7377 sc. to lbs. (18.) 897600 grs. to lbs.

Ex. 17.

Length.

(1.) 150 miles to feet. (2.) 3 miles 6 fur. to in.
(3.) 64 m. 3 fur. 4 yds. to in. (4.) 213 m. 5 fur. 10 po. to ft.
(5.) 177 m. 1025 yds. to ft. (6.) 2716 yds. 9 in. to in.
(7.) 165 po. 8 in. to in. (8.) 86 lea. 8 po. to in.
(9.) 11 lea. 1 fur. 174 yds. to ft. (10.) 55 fur. 27 po. 2½ yds. to ft.
(11.) 84 m. 2 fur. to chains. (12.) 4 m. 26 ch. 96 lk. to lks.

(13.) 396000 yds. to miles. (14.) 4809 in. to yards.
(15.) 57000 in. to fur. (16.) 550462 yds. to miles.
(17.) 17080 po. to lea. (18.) 324160 ft. to lea.
(19.) 32757120 in. to miles. (20.) 578600 in. to poles.
(21.) 1000000 ft. to miles. (22.) 253440 in. to miles.
(23.) 2240 chains to miles. (24.) 270936 links to miles.

(25.) 27 yds. 3 qrs. 2 nls. to nls. (26.) 11 yds. 1 qr. 3 nls. to in.
(27.) 3070 yds. to E. ells. (28.) 225 E. ells to Fl. ells.
(29.) 228 Fr. ells to Fl. ells. (30.) 342 yds. to Fr. ells.
(31.) 675 E. ells to in. (32) 7945 yds. to E. ells.

Ex. 18.

Capacity.

Reduce

(1.) 516 bus. 3 pks. to pks. (2.) 26 bus. 2 pks. 1 gal. to gals.
(3.) 237 quarters to pints. (4.) 22 pks. 3 qts. 1 pt. to pints.
(5.) 91 gals. 3 qts. 1 pt. to gills (6.) 27 qrs. 7 bus. 1 gal. to qts.
(7.) 2319 qrs. 5 bus. 3 pks. 1 gal. 3 qts. 1 pt. to gills.
(8.) How many pints in 4 hogsheads, each 54 gals. 3 qts.?

(9.) 279 pts. to pks. (10.) 1608 qts. to qrs.
(11.) 1371 pks. to qrs. (12.) 30005 gills to gals.
(13.) 5279 pts. to bus. (14.) 71698 qts. to qrs.
(15.) 1000000 pts. to gals. (16.) 737468 pts. to qrs.

Ex. 19.

Time.

Reduce

(1.) 3 weeks to hours. (2.) 13113 days 16 hrs. to hrs.
(3.) 365 days 6 hrs. to sec. (4.) 3 yrs. 17 wks. 5 days to days.
(5.) 5 com. yrs. 300 days to hrs. (6.) 100 Jul. yrs. to min.
(7.) 1000 com. yrs. to hrs. (8.) 3 co. yrs. 35 wks. 2 dys. to sec.

(9.) 12675 hrs. to months. (10.) 80000 seconds to hours.
(11.) 806400 min. to weeks. (12.) 2551440 seconds to days.
(13.) 31556928 sec. to days. (14.) 1000000 minutes to years.
(15.) In 25 *solar* years and 25 days how many seconds?
(16.) In what time could a person count 666 millions of sovereigns; reckoning 6 days in the week, and 12 hours per day, allowing him to count 100 per minute?

Ex. 20.

Surface, Solidity, Inclination.

Reduce

(1.) 628 sq. yds. to inches. (2.) 152 acres to sq. yds.
(3.) 178 ac. 0 ro. 15 po. to po. (4.) 3 ro. 16 po. 13 yds. to ft.
(5.) 646 ac. 0 ro. 16 po. to sq. yds. (6.) 7 sq. miles to poles.
(7.) 30 cub. yds. 26 ft. to in. (8.) 17 cub. yds. 1001 in. to in.
(9.) 23 c. yds. 17 ft. 620 in. to in. (10.) 155 ac. 21 yds. to sq. ft.
(11.) 2 ac. 0 ro. 10 po. 17 yds. 4 ft. 72 in. to inches.
(12.) 15 ac. 1 ro. 17 po. 27 yds. 5 ft. 57 in. to inches.

(13.) 23492 sq. in. to yds.
(14.) 224000 ac. to sq. miles.
(15.) 117892 po. to acres.
(16.) 16214 sq. yds. to po.
(17.) 196992 cub. in. to yds.
(18.) 276829 cub. in. to yds.
(19.) 291852 sq. ft. to po.
(20.) 1000000 sq. in. to ro.
(21) 34677 sq. yds. to ac.
(22.) 1000000 cub. in. to yds.

(23) 44° 13′ 39″ to sec.
(24.) 97° 43′ 58″ to sec.
(25.) 239543 sec. to deg.
(26.) 84457 sec. to deg.

(27.) How many square yards in a court yard 376 feet long and 169 feet broad?

(28.) How many acres are there in a square, each side of which is 660 feet?

(29.) How many cubic inches in a block of stone, 9 feet long, 4 feet broad, and 2 feet thick?

(30.) How many cubic yards in a trough 27 feet long, 13 feet broad, and 5 feet deep?

Ex. 21.

Metric Tables.

The reduction of quantities on the Metric System is simpler than on the Imperial System; the multiplier or divisor being always 10 or some power of 10.

Ex. 1.—How many mètres in 56 kilom. 4 hectom. 5 decam. 3 met.?

$$56000 + 400 + 50 + 3 = 56453 \text{ mètres.}$$

Ex. 2.—How many kilogrammes in 12005 grammes?

$$12005 \div 1000 = 12 \text{ kilog. } 5 \text{ grammes.}$$

Ex. 3.—Reduce 8760105 centilitres to hectolitres.

$$8760105 \div 10000 = 876 \text{ hectolit. } 1 \text{ litre, } 5 \text{ centilit.}$$

Reduce

(1.) 50 kilom.; 71 kilom. 5 hectom. 3 m.; 89 myriam. 7 kilom. 3 m.; 76431 millimètres; to mètres.

(2.) 12 kilog. 67 grammes; 961 kilog. 572 grammes; 518764 milligrammes; to grammes.

(3.) 23 hectares, 9 ares, 35 centiares; 128 hectar. 61 ares; to centiares.

(4.) 17 kilolitres, 3 decalit.; 31 kilolit. 7 hectolit. 4 lit.; to litres.

(5.) 3 quintals, 69 kilog. to grammes; 109 quintals to decagrammes; 56 kilog. to centigrammes.

(6.) 9620074 litres; 1096745 decilit.; 100 decalit. 7 lit. to hectolitres.

(7.) 3476512 milligr. to grms. (8.) 9348267 centim. to mètres.
(9.) 5201001 milligr. to kilog. (10.) 8967439 decilit. to kilolit.
(11.) 437892 centiares to hectar. (12.) 6200706 grms. to quintals.
(13.) 8900172 centigr. to kilog. (14.) 8710009 centim. to kilom.
(15.) 5030001 decilit. to hectolit. (16.) 4059008 decigr. to quintals.

VII.—COMPOUND ADDITION.

Compound Addition is the adding together into one sum of several numbers of different denominations of the same kind.

Rule.—Set the numbers or quantities to be added together under one another, so that the parts which are of *the same denomination* may stand *in the same column*, and draw a line below them.

Add together the numbers of the lowest denomination (those in the right hand column), and find, by reduction, how many of the next denomination are contained in this sum.

Set down the remainder, if any, under the column added, and *carry* (that is *add*) the quotient to the next column; add this up as before and so on with all the columns.

Example 1.—

£	s.	d.
57	15	6
22	3	10¼
15	8	4½
49	16	1¾
87	10	0¼
232	13	10¾

Worked thus:—1 farthing + 3 farthings + 2 farthings + 1 farthing = 7 farthings = 1 penny and 3 farthings. *Put down* ¾ under the farthings *and carry* 1 penny to the pence column.

(1 + 1 + 4 + 10 + 6)d. = 22 pence = 1 shilling and 10 pence. *Put down* 10 under the pence *and carry* 1 shilling to the shillings' column.

(1 + 10 + 16 + 8 + 3 + 15)s. = 53 shillings = 2 pounds and 13 shillings. *Put down* 13 under the shillings, *and carry* £2 to the pounds' column.

(2 + 7 + 9 + 5 + 2 + 7) pounds = £32. *Put down* 2 under the units of pounds, *and carry* 3 (tens) to the tens' column.

3 + 8 + 4 + 1 + 2 + 5 = 23. *Put down* 23 (tens).

Answer, £232 13s. 10¾d.

EXAMPLE 2.—

tons	cwt.	qrs.	lbs.	oz.
2	15	3	12	8
4	8	0	17	6
7	9	1	20	3
8	11	2	6	12
6	2	3	12	7
29	7	3	13	4

7 + 12 + 3 + 6 + 8 = 36 oz.; 36 ÷ 16 = 2 lbs. 4 oz.
2 + 12 + 6 + 20 + 17 + 12 = 69 lbs.; 69 ÷ 28 = 2 qrs. 13 lbs.
2 + 3 + 2 + 1 + 0 + 3 = 11 qrs.; 11 ÷ 4 = 2 cwt. 3 qrs.
2 + 2 + 11 + 9 + 8 + 15 = 47 cwt.; 47 ÷ 20 = 2 tons 7 cwt.
2 + 6 + 8 + 7 + 4 + 2 = 29 tons.

PROOF.—As in Simple Addition.

Ex. 22.

(1.)	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
	15	17	10	53	14	8	4	0	7¼	37	15	8
	3	14	6	5	10	2¾	15	13	6	14	12	9¾
	23	6	2¾	93	11	6	2	4	7	5	6	11
	8	3	5	7	5	0¼	64	17	8	23	10	9¼
	15	6	4	2	0	9½	3	0	4¾	8	6	0
	6	12	9¾	0	18	7	26	6	7	14	0	5½

(2.)	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
	61	3	2½	472	15	3	27	13	3	14	7	5
	7	16	8	9	2	2½	3	5	10½	8	19	2¼
	29	13	10¾	27	12	6¼	6	18	7	5	3	4½
	8	14	0	370	16	2½	20	2	5¾	21	2	9¼
	10	7	5¼	25	3	8	4	0	3	7	16	8½
	24	13	0	6	10	5¾	17	15	4½	30	4	3
	5	0	10¾	30	0	11¾	91	0	10¼	54	2	7½

(3.)	£	s.	d	£	s.	d.	£	s.	d.	£	s.	d.
	496	18	10¼	476	19	10¾	976	18	11¾	479	18	9¾
	57	19	7½	47	18	9¼	976	18	10½	976	14	8¼
	37	18	7½	42	18	9¼	432	19	10½	1430	12	10¼
	14	9	8¾	432	18	7½	40	19	4¼	1402	18	2
	423	5	2¼	47	17	1¼	496	18	4¾	4942	17	10¼
	232	14	10¾	476	1	6¾	987	10	10¾	983	10	6¾

£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
978	7	1¼	374	18	7¾	876	13	2½	579	12	6¼
942	18	7¼	972	18	7½	231	10	8	338	14	5¾
42	8	7¾	332	19	7¼	548	7	11¼	296	17	8½
769	18	7¼	574	18	9¼	917	16	9	638	16	9¼
947	8	10½	9768	18	2½	803	1	0¾	579	13	5¼
654	18	1¼	137	2	4½	919	19	1½	867	11	2
322	19	7¼	936	1	10¾	407	18	9	993	16	8¾

£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
779	9	6¾	705	6	7½	299	14	8¼	27	8	8¼
36	17	8½	95	6	9¾	323	6	7½	796	14	5½
9	12	7¾	282	17	3¾	763	4	9¾	75	16	7¼
784	15	6¼	1	19	8½	65	8	6¾	63	14	6¾
269	14	9¾	76	14	6¾	29	16	8½	83	16	9¼
24	15	3¾	197	17	9½	36	17	4¾	206	14	7½
289	14	6¾	57	6	8¾	92	18	10¼	43	12	6½
23	16	8¾	87	16	8½	276	14	6½	79	15	7¼
708	15	8½	2	14	3¼	16	9	7¾	287	16	6¾

£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
212	18	9	24	6	10¼	28	13	5½	118	5	8¾
133	3	10¼	77	12	11¾	108	12	6¾	58	16	5½
94	15	5¾	29	14	9¾	57	17	6½	26	15	7¾
87	16	6½	66	19	4½	69	16	7¼	139	14	6¾
101	17	7½	78	19	8½	103	14	8¾	15	13	11½
57	18	8¾	75	12	3¾	78	12	7¼	125	17	6½
84	0	9½	67	6	11½	69	19	6¾	69	19	7¼
98	19	0¾	46	9	6¾	79	12	5¼	76	16	6¾
56	17	11	38	17	8½	87	14	6½	66	14	7¼

£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
324	9	6¼	797	16	9½	969	16	8¼	326	17	6¾
777	7	7¼	62	19	8¼	99	7	7¾	69	18	7½
689	8	6¾	226	14	9¾	76	6	6¾	807	7	9¾
26	18	7¾	379	16	6¼	36	13	5¼	275	19	7½
39	12	6¼	25	13	8¾	248	16	6¾	34	17	4½
667	19	8¾	61	17	8½	675	12	4½	396	19	11¾
736	18	4½	707	14	9¾	267	6	7¾	61	17	4¾
894	17	8½	274	16	8¾	484	7	6¾	298	12	11½
767	*16*	*9¾*	*626*	*16*	*6½*	26	18	8¾	84	6	4¼

(8.)

£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
167	9	9	143	15	8¼	191	11	7½	257	12	9¼
846	17	8	468	19	0½	416	13	9½	742	7	2¾
783	14	5	948	0	8¼	963	15	7¾	416	8	11½
412	11	4	146	11	10½	648	19	9½	837	13	3¼
971	15	7	148	11	9¼	146	13	4½	583	11	0½
149	17	8	937	10	10¾	163	17	8½	162	6	8¾
416	11	3	719	11	10½	129	17	11¼	672	12	9½
879	18	4	786	11	4½	100	17	4½	247	18	4¼
358	13	8	687	15	9¾	192	11	11	752	1	7¾

(9)

£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
4716	12	3½	3846	17	8½	7259	11	5¼	6345	3	4½
5383	8	11¼	2754	2	4¾	3374	2	4	2687	1	5¼
5360	15	7¾	3680	15	3¼	3170	15	7	9974	12	7½
7629	6	6½	2753	10	7½	5428	9	7½	7112	18	6¾
3237	8	4¼	6168	16	2	4615	11	9¾	1033	7	11
6853	15	8½	7312	11	10¼	6377	8	3	2528	17	9¼
2036	7	7¼	5689	9	2	2561	4	2	8456	19	3
4761	11	0½	6245	4	4¾	8434	0	9¼	5391	8	1¾
3237	4	3¾	4317	10	7½	7829	10	4½	6754	2	0¼
5948	13	5½	5028	9	6¾	3625	8	11¾	4608	16	11
6162	11	8½	4971	3	5½	1740	17	7½	3245	11	1

(10.) £79 13s. 9½d. + £113 3s. 8¼d. + £7008 15s. 4d. + £36 14s. 7½d. + £1065 19s. 5¾d. + £978 12s. 4d.

(11.) £10 7s. 3d. + £234 15s. 6d. + £432 10s. 0¼d. + £308 15s, 4¼d. + £265 7s. 4¼d. + £800 19s. 6d. + £731 10s. 10d.

(12.) £593 19s. 4½d. + £855 13s. 6¼d. + £818 1s. 4½d. + £952 11s. 7¼d. + £768 17s. 8¾d. + £587 18s. 9½d. + £113 12s. 8¼d. + £916 10s. 0¾d.

Ex. 23.

(1.)

cwt.	qrs.	lbs.	cwt.	qrs.	lbs.	cwt.	qrs.	lbs.	cwt.	qrs.	lbs.
15	2	15	99	3	7	29	3	20	32	3	15
6	3	24	17	1	11	43	2	17	17	2	19
7	0	10	6	0	27	8	1	13	43	2	18
9	1	17	14	2	19	14	0	19	24	1	27
10	2	6	17	1	12	27	3	15	26	2	23
3	0	3	29	0	24	14	3	18	35	3	21

)

tons	cwt.	qrs.	tons	cwt.	qrs.	lbs.	oz.	drs.	lbs.	oz.	drs.
4	7	1	22	13	1	23	12	8	25	10	12
3	13	2	34	5	2	4	10	9	13	14	12
2	18	3	16	17	0½	13	15	12	12	12	13
1	15	2	19	18	3¼	5	1	3	43	15	12
2	14	3	28	2	1	14	13	7	14	11	15
3	19	3	10	19	3¾	13	6	13	6	14	12

)

oz.	dwt.	grs.	oz.	dwt.	grs.	lbs.	oz.	dwt.	lbs.	oz.	dwt.
17	13	18	91	12	19	29	5	18	36	7	13
25	16	12	89	17	14	32	7	11	54	4	19
31	19	4	70	21	13	47	9	16	99	9	17
8	10	23	18	13	21	26	11	19	87	8	9
10	0	20	72	0	12	34	5	17	76	6	7
27	16	21	49	17	16	87	8	12	39	9	8

)

drs.	sc.	grs.	drs.	sc.	grs.	oz.	drs.	sc.	oz.	drs.	sc.
12	1	14	35	0	15	40	6	1	15	0	2
27	2	5	42	1	19	35	7	2	19	7	1
13	0	18	73	2	11	81	0	0	7	3	0
37	1	19	61	0	17	27	1	1	11	6	2
46	0	11	17	2	18	32	6	2	12	4	1
24	2	9	86	1	19	14	5	0	26	5	2

)

yds.	qrs.	nls,	yds.	qrs.	nls.	yds.	ft.	in.	yds.	ft.	in
5	2	1	8	3	1	16	2	9	126	1	9
6	1	3	5	1	2	17	1	11	37	0	11
7	1	3	13	0	1	18	0	5	103	2	8
8	2	1	5	0	3	17	2	7	·46	1	10
9	3	3	6	2	2	12	1	8	234	0	10
7	3	2	7	2	2	14	0	9	119	2	9

)

po.	yds.	ft.	fur.	po.	yds.	miles	fur.	po.	miles	fur.	po.
27	4	1	7	29	5	3	2	19	29	3	14
32	5	2	5	36	0	6	1	16	19	6	29
15	3	0	6	37	4	7	1	18	5	4	20
16	1	2	3	14	3	8	2	15	9	1	37
14	5	1	4	27	4	9	2	39	7	0	3
37	4	1	5	16	2	7	7	17	4	5	9

(7.)	gal.	qts.	pts.	gal.	qts.	pts.	qrs.	bu.	gal.	qrs.	bu.	[illegible]
	13	2	1	43	2	1	3	0	0	5	3	[illegible]
	21	3	1	9	1	0	24	3	7	6	2	[illegible]
	12	3	0	16	3	0	41	7	4	25	7	[illegible]
	10	1	0	8	0	1	13	5	2	17	5	[illegible]
	21	3	1	13	2	0	7	1	3	7	4	[illegible]
	23	0	1	6	3	1	5	2	1	33	0	[illegible]

(8.)	ac.	r.	po.	ac.	r.	po.	sq. yds.	ft.	in.	sq. yds.	ft.	in.
	16	1	25	75	0	23	172	4	108	712	3	141
	9	0	23	19	0	16	86	3	97	864	8	109
	225	3	27	9	1	3	435	5	116	36	1	17
	4	2	9	7	2	16	599	7	39	157	7	28
	7	0	6	23	0	34	78	0	47	97	4	136
	42	1	19	270	3	29	36	8	67	111	1	111

(9.)	ro.	po.	sq. yds.	po.	sq. y.	ft.	cu. yds.	ft.	in.	cu. yds.	ft.	in.
	23	39	29¼	27	27	6¼	47	13	1610	17	16	579
	2	19	17½	36	19	3	35	17	578	19	19	1658
	75	14	30	57	28	4½	67	26	1001	28	21	786
	3	29	15¼	19	27	5	89	24	969	36	18	1291
	18	17	27	16	15	8¼	97	19	1537	49	23	367
	99	31	19	29	12	0	69	17	1299	58	16	898

(10.)	dys.	hrs.	min.	hrs.	min.	sec.	wks.	dys.	hrs.	yrs.	dys.	hrs.
	27	23	58	23	59	59	48	3	19	9	315	22
	13	6	37	25	18	47	36	0	23	11	108	19
	19	18	49	17	17	48	51	5	17	23	296	21
	38	9	56	13	39	19	17	2	8	19	189	18
	96	27	19	21	18	27	39	4	16	18	353	17
	48	14	37	35	14	49	28	1	22	57	199	23

Add together:—

(11.) 63 cwt. 3 qrs. 14 lbs. 5 oz.; 562 cwt. 1 qr. 19 lbs. 12 oz.; 916 cwt. 0 qr. 21 lbs. 17 oz.; 188 cwt. 3 qrs. 15 lbs. 13 oz.; 956 cwt. 2 qrs. 25 lbs. 11 oz.; 764 cwt. 1 qr. 11 lbs. 2 oz.; 981 cwt. 3 qrs. 17 lbs. 15 oz.; 858 cwt. 1 qr. 22 lbs. 9 oz.

(12.) 8 lbs. 6 oz. 4 dwts. 20 grs.; 1 lb. 10 oz. 5 dwts. 12 grs.; 24 lbs. 10 oz. 14 dwts. 19 grs.; 24 lbs. 8 oz. 13 dwts. 21 grs.; 22 lbs. 17 dwts. 15 grs.; 15 lbs. 11 oz. 18 grs.; 34 lbs. 9 oz. 12 dwts.; 74 lbs. 11 oz. 16 dwts. 9 grs.; 17 oz. 9 dwts. 19 grs.

(13.) 10 lbs. 8 oz. 4 drs. 1 scr.; 66 lbs. 10 oz. 2 drs.; 19 lbs. 9 oz. drs. 2 scr.; 55 lbs. 6 drs.; 79 lbs. 11 oz. 4 drs. 1 scr.; 13 lbs. 6 oz. drs. 2 scr.

(14.) 693 yds. 3 qrs. 2 nls.; 605 yds. 2 qrs. 1 nl.; 854 yds. 3 qrs. 2 ls.; 739 yds. 2 qrs. 3 nls.; 379 yds. 2 qrs. 1 nl.; 973 yds. 3 qrs. 1 nl.

(15.) 495 yds. 2 ft. 9 in.; 953 yds. 1 ft. 8 in.; 359 yds. 2 ft. 11 in.; 97 yds. 1 ft. 6 in.; 967 yds. 2 ft. 8 in.; 796 yds. 1 ft. 7 in.

(16.) 137 qrs. 5 bus. 13 pts.; 243 qrs. 2 pks. 14 qts.; 106 qrs. 4 bus. 3 pts.; 137 qrs. 7 bus. 2 gals.; 92 qrs. 4 bus. 17 pks.

(17.) 55 hhds 33 gals 2 qts.; 161 hhds. 24 gals. 1 qt.; 16 hhds. 3 gals. qts.; 161 hhds. 45 gals. 1 qt.; 776 hhds. 51 gals. 2 qts.; 1677 hhds. 34 als 3 qts.; 767 hhds. 47 gals. 2 qts; 616 hhds. 49 gals. 2 qts.; 947 hds. 56 gals 1 qt.; 1497 hhds. 54 gals. 3 qts.; 947 hhds. 19 gals. 2 qts.; 94 hhds. 36 gals. 2 qts.; 794 hhds. 36 gals. 2 qts.: 974 hhds. 13 gals. 2 ts.; 2479 hhds. 51 gals. 1 qt.; 4 hhds. 22 gals. 1 qt.

N.B. 63 gals. = 1 hhd.

(18.) 271 acres, 2 roods, 32 perches; 261 a. 1 r. 15 p.; 19 a. 3 r. 9 p.; 721 a. 1 r. 12 p.; 182 a. 3 r. 36 p.; 119 a. 0 r. 5 p.; 121 a. r. 31 p.; 1 a. 2 r. 3 p.; 119 a. 3 r. 27 p.; 915 a. 1 r. 22 p.; 81 a. r. 26 p.; 97 a. 3 r. 19 p.; 193 a. 2 r. 2 p.; 266 a. 9 r. 30 p.

(19.) 16 mo. 2 wks. 1 h.; 1 wk. 6 d. 23 h. 59 m.; 11 mo. 1 wk. 8 m.; 17 yrs. 28 wks. 3 d.; 26 yrs. 5 wks. 5 d.; 158 yrs. 6 d. 23 h. 9 s.

(20.) 441 yrs. 9 mo. 2 wks. 5 d.; 74 yrs. 7 mo. 3 wks. 6 d.; 48 yrs. 0 mo. 2 wks. 4 d.; 55 yrs. 11 mo. 2 wks. 6 d.; 44 yrs. 7 mo. 3 wks. d.; 47 yrs. 2 mo. 2 wks. 2 d.; 95 yrs. 6 mo. 3 wks. 5 d.; 59 yrs. mo. 2 wks. 3 d.; 49 yrs. 6 mo. 3 wks. 4 d.

VIII.—COMPOUND SUBTRACTION.

COMPOUND SUBTRACTION is the process of finding the *difference* between two numbers of different denominations of the same kind.

RULE.—Set the smaller number under the other, so that the parts which are of the same denomination may be in the same column, and draw a line below them.

Begin at the right hand, and subtract, *if possible*, each number in the lower line from that above it, and set down the remainder.

If the lower number is *greater* than the upper, first add to the upper as many units of that denomination as make one of the next higher denomination,—then subtract the lower number,—set down the remainder,—and carry one to the next number in the lower line; proceed in the same way through the several denominations.

Ex. 1.—

£	s.	d.
27	8	6½
13	12	7¼
13	15	11¼

Worked thus:—1 farthing from 2 farthings = 1 farthing. *Put down* 1 farthing.

You cannot take 7 pence from 6 pence, so *borrow* 12d. (that is 1s.), and add to the 6d., which will make 18d., then 7 from 18 leave 11. *Put down* 11d. *and carry* 1s. (= 12d., borrowed) to the shillings' column.

12 + 1 = 13. Thirteen shillings from 8 shillings you cannot, so *borrow* 20s. (that is £1) and add to the 8s. which will make 28s.; then 13 from 28 leave 15. *Put down* 15s. *and carry* £1 (= 20s. borrowed) to the pounds' column.

3 + 1 = 4. Four pounds from 7 pounds = 3 pounds. *Put down* £3 under the units of pounds.

One (ten) from 2 (tens) = 1 (ten). *Put down* 1 under the tens.

Answer; £13 15s. 11¼d.

Ex. 2.—

cwt.	qrs.	lbs.	oz.
47	3	19	4
26	2	24	12
21	0	22	8

Worked thus:—12 from 20 (4 + 16) = 8. *Put down* 8 oz. *and carry* 1 lb. (= 16 oz. borrowed).

24 + 1 = 25. 25 from 47 (19 + 28) = 22. *Put down* 22 lbs. *and carry* 1 qr. (= 28 lbs. borrowed.)

2 + 1 = 3. 3 from 3 = 0. *Put down* 0 qrs.

26 from 47 = 21. *Put down* 21 cwt.

Answer. 21 cwt. 0 qrs. 22 lbs. 8 oz.

Proof.—Add the *remainder* and *subtrahend* together, the result should be the *minuend* or larger number.

Ex. 24.

s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
16	7½	96	14	6¾	74	9	0¼	27	11	11¾
2	5¼	49	13	8½	59	12	10½	21	19	7¼

s	d.	£	s.	d.	£	s.	d.	£	s.	d.
2	6½	94	15	7¾	200	1	6¼	261	7	8¼
7	9¾	89	18	8¼	99	18	3½	119	15	10¾

s	d.	£	s.	d.	£	s.	d.	£	s.	d.
3	7¼	465	7	3	852	10	7¾	999	19	6¼
7	9½	258	14	6¾	568	9	11½	573	19	7¾

s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
5	1½	527	3	5	639	13	10¾	861	17	5¼
6	11¾	139	5	7¼	587	17	10½	387	18	8¾

s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
3	3¼	621	10	2	716	1	0½	917	3	2½
4	5½	399	15	8¾	568	13	7¼	436	15	10¾

s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
8	3	8965	4	7½	9647	16	2¼	4917	0	0½
9	5¼	5098	17	9¾	5789	17	6	3899	11	7¾

97 8s. 11d. — £3180 11s. 2½d.; £2852 0s. 7¾d. — £764

350 6s. 5¾d. — £47541 5s. 6¾d.; £51862 11s. 1½d. —
s. 9¾d.
14s. 6¼d. — £14 8s. 5½d. + £23 6s. 7½d. + £46 5s. 3¼d.
11d. — £39 6s. 9¾d.
163 10s. 7d. + £2178 8s. 11d. — £7648 13s. 1¼d. + £5896
- £3178 15s. 9½d
e receipts of an exhibition are £5006 11s. 6d. and the ex-
l £1167 15s. 9d.; what are the profits?
at is the difference between 495 guineas and £387 12s. 8¼d.
nan earns 19s. a week, his wife 6s. 9d., one of his two sons
the other 3s. 9d. They pay 4s. 6d. a week for rent; how
they for food, clothing, &c.?

(14.) I have £97 15s. 7½d. in my pocket, what will be left after paying the following bills :—£14 18s. 6d.; £29 3s. 10d.; £21 19s. 9d.; and £6 15s. 4d.?

Ex. 25.

(1.)	cwt.	qrs.	lbs.	cwt.	qrs.	lbs.	cwt.	qrs.	lbs.	cwt.	qrs.	lbs.
	23	0	11	27	2	3	206	2	14	357	1	18
	18	2	17	12	3	22	18	0	21	179	3	23
(2.)	lbs.	oz.	drs.	lbs.	oz.	drs.	tons	cwt.	qrs.	tons	cwt.	qrs
	54	10	7	42	3	11	47	11	1	85	9	2
	38	14	12	27	14	13	29	17	3	67	11	3
(3.)	oz.	dwt.	grs.	oz.	dwt.	grs.	lbs.	oz.	dts.	lbs.	oz.	dts.
	7	15	2	214	10	17	11	2	13	16	11	15
	6	18	23	106	19	20	8	9	15	12	3	19
(4.)	drs.	sc.	grs.	drs.	sc.	grs.	oz.	drs.	sc.	oz.	drs.	sc.
	32	1	11	47	0	8	247	3	1	318	5	0
	17	2	18	29	1	15	129	7	2	159	6	1
(5.)	yds.	qrs.	nls.	yds.	qrs.	nls.	yds.	ft.	in.	yds.	ft.	in.
	27	3	0	68	1	1	176	1	9	471	2	5
	18	0	3	47	3	2	119	2	11	289	1	7
(6.)	po.	yds.	ft.	fur.	po.	yds.	miles	fur.	po.	miles	fur.	yds.
	27	4	1	7	15	4	39	5	22	23	5	193
	19	1	2	4	38	3½	12	7	29	19	7	204
(7.)	lea.	mi.	ft.	mi.	yds.	ft	E. ells	qrs.	nls.	F. ells	qrs.	nls.
	17	2	1100	98	216	1	217	4	1	567	1	0
	11	1	1540	76	549	2	109	2	2	389	5	1
(8.)	gal.	qts.	pts.	gal.	qts.	pts.	qrs.	bu.	pks.	qrs.	bu.	gal.
	271	1	1	631	2	0	53	7	1	97	3	3
	197	3	1	365	3	1	42	6	3	49	7	6
(9.)	ac.	r.	po.	ac.	r.	po.	sq. yds.	ft.	in.	sq. yds.	ft.	in.
	76	1	28	98	0	17	516	3	97	16	8	73
	24	2	37	69	3	22	287	5	118	6	8	121

(10.)	ro.	po.	sq.yds.	po.	sq.y.	ft.	cu.yds.	ft.	in.	cu.yds.	ft.	in.
	3	30	27	29	28	7	371	4	327	73	1	954
	1	37	14	17	15	8	159	19	1011	7	25	1000

(11.)	dys.	hrs.	min.	hrs.	min.	sec.	wks.	dys.	hrs.	yrs.	dys.	hrs.
	29	17	31	17	23	30	53	5	17	797	3	15
	17	19	54	14	49	52	14	6	23	379	11	23

(12.)	yrs.	wks.	dys.	yrs.	dys.	hrs.	deg.	min.	sec.	deg.	min.	sec.
	492	37	4	77	162	12.	66	32	33	78	14	16
	189	48	6	39	287	19	33	17	47	29	27	31

Find the difference between :—

(13.) 19 cwt. 2 qrs. 12 lbs. and 23 cwt. 3 qrs. 20 lbs.; also between 25 tons 7 cwt. 24 lbs. and 15 tons 12 cwt. 1 qr.

(14.) 10 days, 20 hrs. 54 min. 30 sec. and 4 days 4 hrs. 26 min. 58 sec.; also between 78 wks. and 27 wks. 6 days, 16 hrs. 22 min. 13 sec.

(15.) 687 lbs. 9 oz. 8 grs. and 288 lbs. 1 oz. 9 dwts. 20 grs.; also between 5 lbs. 2 oz. 1 dr. and 2 lbs. 5 oz. 2 dr. 1 sec.

(16.) 79 miles, 4 fur. and 12 miles, 6 fur. 3 yds. 2 ft.; also between 71 lea. 1 mi. 7 fur. and 50 lea. 3 fur. 27 po.

(17.) 780 acres, 2 rds. and 396 ac. 3 rds. 15 p.; also between 325 ac. 2 rds. 1 po. and 379 ac. 3 rds. 5 po.

(18.) 1000 lbs. and 1000 oz. troy; also between 1000 lbs. and 1000 oz. avoirdupois.

(19.) A young man has served 5 yrs. 2 mo. 3 wks. 4 dys. of his 7 years' apprenticeship, how much longer has he to serve?

(20.) The great bell (Great Tom) at Lincoln weighs 4 tons, 16 cwt 2 qrs. 6 lbs., Big Ben at Westminster weighs 13 tons, 10 cwt. 1 qr. 18 lbs., what is the difference in their weights?

IX.—COMPOUND MULTIPLICATION.

Compound Multiplication is the method of finding the amount of any quantity, composed of different denominations *of the same kind,* when repeated or multiplied a given number of times.

Rule:—Place the multiplier under the lowest denomination of the multiplicand; multiply the number of this lowest denomination by the multiplier, and find, by reduction, how many of the next denomination are contained in this product.

Set down the *remainder*, if any, under the number multiplied, and *carry* (i.e. *add*) the quotient to the product obtained by multiplying the next higher denomination; reduce as before and so on through all the denominations. (Ex. 1.)

If the multiplier is *greater* than 12 (or 20 if the extended multiplication table be known) it will be convenient to divide it if possible into factors, or into factors and a remainder (each not greater than 12 or 20) and multiply by each factor *successively*. (Ex. 2, 3.)

Thus 48 = 6 × 8; 125 = 5 × 5 × 5; 112 = 7 × 16; 137 = 11 × 12 + 5.

When the multiplier is *very large* it will frequently be most convenient to reduce the multiplicand to its lowest denomination, then multiply the result by the multiplier and then reduce the product back again. (Ex. 4.)

(1.) Multiply £38 15s. 4½d. by 7.

EXAMPLE:—£	s.	d.
38	15	4½
		7
271	7	7½

Worked thus:—Seven times 2 farthings = 14 farthings = 3½d. *Put down* ½d. *and carry* 3d. to the product of the pence. Seven times 4d. = 28d. 28d. + 3d. = 31d. = 2s. 7d. *Put down* 7d. *and carry* 2s. to the product of the shillings.

Seven times 15s. = 105s. 105s. + 2s. = 107s. = £5 7s. *Put down* 7s. *and carry* £5 to the product of the pounds.

Seven times 38 = 266: £266 + £5 = £271. *Put down* 271.

(2.) Multiply 12 cwt. 1 qr. 17 lbs. by 88 (= 11 × 8).

cwt.	qrs.	lbs.	
12	1	17	
		8	
99	0	24	= 12 cwt. 1 qr. 17 lbs. × 8.
		11	
1091	1	12	= 12 cwt. 1 qr. 17 lbs. × 88.

(3.) Multiply 4 miles, 3 fur. 23 po. by 113 (= 10 × 11 + 3).

m.	f.	p.	
4	3	23	× 3
		10	
44	3	30	= 4 m. 3 f. 23 p. × 10.
		11	
489	1	10	= 4 m. 3 f. 23 p. × 110.
13	2	29	= 4 m. 3 f. 23 p. × 3.
502	3	39	= 4 m. 3 f. 23 p. × 113.

(4.) Multiply 17 gals. 2 qts. 1 pt. by 523.

gals.	qts.	pts.	
17	2	1	× 3
		10	
176	1	0	× 2
		10	
1762	2	0	
		5	
8812	2	0	= 500 times.
352	2	0	= 20 ,,
52	3	1	= 3 ,,
9217	3	1	= 523 ,,

gals.	qts.	pts.	
17	2	1	
4			
70			
2			
141			pints.
523			
423			
282			
705			
2)73743			pints.
4)36871	1		
9127	3	1	

Ex. 26.

	£	s.	d.			£	s.	d.	
(1.)	22	18	3	× 2, 3, 4, 5.	(2.)	37	6	2¼	× 2, 3, 4, 5.
(3.)	24	6	4¼	× 4, 5, 6, 7.	(4.)	69	10	2½	× 4, 5, 6, 7.

	£	s.	d.			£	s.	d.	
(5.)	97	11	2½	× 5, 6, 7, 8.	(6.)	58	11	7	× 7, 8, 9, 10.
(7.)	89	15	6¾	× 7, 8, 9, 10.	(8.)	47	13	3¾	× 8, 9, 11, 12.
(9.)	76	10	9¼	× 9, 10, 11, 12	(10.)	99	18	10½	× 7, 9, 11, 12.

	£	s.	d.			£	s.	d.	
(11.)	36	11	3¼	× 27, 36, 63.	(12.)	48	13	6½	× 54, 64, 84.
(13.)	20	17	9½	× 22, 32, 42.	(14.)	57	16	8¾	× 21, 81, 121.
(15.)	85	8	7¾	× 36, 56, 96.	(16.)	89	17	5½	× 48, 88, 108.
(17.)	98	11	9¾	× 49, 99, 132.	(18.)	97	9	11¼	× 72, 110, 144.

	£	s.	d.			£	s.	d.	
(19.)	376	11	3	× 23, 29, 37.	(20.)	458	9	4¼	× 38, 57, 69.
(21.)	497	13	2½	× 97, 103, 115.	(22.)	519	11	8¾	× 47, 83, 91.
(23.)	501	0	11¾	× 92, 94, 59.	(24.)	699	13	2¼	× 48, 61, 76.

	£	s	d.			£	s.	d.	
(25.)	678	16	10	× 139, 151.	(26.)	857	5	4¼	× 143, 217.
(27.)	538	14	7¼	× 336, 405.	(28.)	289	7	9	× 652, 796.
(29.)	697	2	10½	× 847, 954.	(30.)	776	18	7¼	× 5683, 7945.

Find the value of

(31.) 26 articles at £13 13s. 3¼d. each; of 49 yds. at £1 10s. 9d. per yd.

(32.) 59 cwt. at £2 3s. 4d. per cwt.; 79 oz. at £3 14s. 10½d. per oz.

(33.) 125 gals. at £3 5s. 7d. per gal.; 343 qrs. at £2 17s. 9d. per qr.

(34.) 588 lbs. at £3 12s. 6½d. per lb.; 648 lbs. at £3 11s. 8d. per lb.

(35.) 432 yds. at £1 2s. 6d. per yd.; 513 yds. at £1 7s. 4d. per yd.

(36.) 517 acres at £4 3s. 6d. per acre.; 496 acres at £5 8s. 9d. per acre.

Ex. 27,

(1.) 17 lbs. 7 oz. 8 dwts. 9 grs. × 6, 9, 49, 55, 64.
(2.) 13 cwt. 1 qr. 12 lbs. 3 oz. × 9, 15, 24, 70, 81.
(3.) 22 cwt. 2 qrs. 6 lbs. 6 oz. × 4, 6, 27, 48, 63.
(4.) 37 cwt. 0 qrs. 15 lbs. × 11, 16, 72, 85, 100.
(5.) 39 yds. 3 qrs. 2 nls. 2 in. × 14, 19, 36, 59, 87.
(6.) 17 miles, 7 fur. 11 po. × 18, 37, 45, 53, 91.
(7.) 7 ac. 0 r. 38 po. 17 yds. × 6, 10, 23, 38, 210.
(8.) 217 gals. 3 qts. 1 pt. × 8, 9, 17, 39, 105.
(9.) 148 gals. 2 qts. 1 pt. × 21, 32, 43, 55, 117.
(10.) 21 hrs. 31 min. 49 sec. × 17, 108, 225, 317, 479.
(11.) 29 dys. 12 hrs. 44 min. × 19, 58, 67, 146, 537.
(12.) 2 fur. 19 yds. 2 ft. 8 in. × 28, 45, 79, 215, 729.

(13.) What is the weight of 27 coins, each 10 dwts. 18 grs.?

(14.) What is the weight of 68 packages, each 25 lbs. 8 oz.?

(15.) What distance will a man travel in 89 days, at the rate of 23 miles, 3 fur. 150 yds. per day?

(16.) What is the weight of 653 cubic feet of water, when one cubic foot weighs 62 lbs. 8 oz.?

(17.) What is the entire length of 179 pieces of cloth, each 27 yds. 3 qrs.?

(18.) Sound travels at the rate of 1 fur. 144 yds. 1 ft. per second; how far will it have reached in 43 seconds?

X.—COMPOUND DIVISION.

Compound Division is the method of dividing a quantity composed of different denominations *of the same kind* into a given number of *equal* parts, and also of finding *how often* one compound quantity is contained in another *of the same kind.*

1. *When the divisor is an abstract number.*

Rule.—Place the divisor on *the left hand* of the dividend, divide the *highest* denomination of the dividend by it, as in Simple Division; and write down the quotient. Reduce the remainder (if any) to the *next lower* denomination, *adding to it* the number of that denomination in the dividend. Repeat the division and proceed as before.

2. *When the divisor is a concrete number.*

Rule.—Reduce *both* divisor and dividend to the *same name* and divide as in Simple Division.

Example.—Divide £295 16s. 11¼d. by 9.

	£	*s.*	*d.*
9)	295	16	11¼
	32	17	5¼

Worked thus:—9 in 29, 3 and 2 remaining. *Put down* 3 (that is 3 tens of pounds) *and carry* 2. 9 in 25, 2 and 7 remaining. *Put down* 2 (pounds) *and carry* 7 pounds.*

Reduce the £7 16s. to shillings, 140s. + 16s. = 156s.

9 in 15, 1 and 6 remaining; 9 in 66, 7 and 3 remaining. *Put down* 17s. *and carry* 3s.

3s. = 36d.; 36d. + 11d. = 47d.

9 in 47, 5 and 2 remaining. *Put down* 5d. *and carry* 2d.

2d. = 8 farthings. 8 f. + 1 f. = 9 f.

9 in 9, one. *Put down* 1 farthing.

Answer.—£32 17s. 5¼d.

---

* The *form of words* generally used, as above, in describing the work is not correct; for when it is said 9 in £25, 2 and 7 remaining, what are the 2 and the 7? Certainly £2 and £7. But how can 9 be contained in £25, 2 *pounds' times?* the expression is meaningless, still as the result is always correct, the incorrect form of expression may pass. It might, if wished, be *avoided by saying the ninth part* of £25 is £2 and £7 remaining.

Example 2.—Divide 401 cwt. 3 qrs. 4 lbs. by 72; 431 gals. 1 0 pt. by 150; and 90 miles, 1 fur. 28 po. by 47.

```
        cwt. qrs. lbs.
72 { 9)401   3   4
   { 8) 44   2  16
         5   2   9

        gal. qt. pt.
    (10)431   1   0
150{  3) 43   0   1
    (  5) 14   1   1
          2   3   1
```

```
     m.  f.  po.
 47)90   1   28(1 m.
    47
    43
     8
 47)345(7 f.
    329
     16
     40
 47)668(14 po.
    47
    198
    188
     10
      5½
 47) 55(1 yd.
     47
      8/47 of a yard.
```

Example 3.—How many payments of £5 16s. 7d. each ca made out of £64 2s. 5d.?

```
 £  s.  d.
 5 16   7
  20
 116
  12
1399
```

```
         £   s. d.
        64   2  5
         20
       1282
         12
1399)15389(11 payments.
      1399
       1399
       1399
```

Ex. 28.

£ s. d.

(1.) 563 10 8 ÷ 2, 4, 6, 8.

(2.) 479 13 4 ÷ 3, 5, 7, 9

(3.) 679 13 7¼ ÷ 4, 5, 6, 7.

(4.) 257 15 3¼ ÷ 6, 7, 8, 9.

(5.) 459 11 9¾ ÷ 7, 8, 9, 10.

(6.) 899 9 11½ ÷ 5, 7, 9, 11

(7.) 761 17 11½ ÷ 10, 11, 12.

(8.) 780 13 9¾ ÷ 10, 11, 12

	£	s.	d.			£	s.	d.	
(9.)	278	18	4¼	÷ 16, 18, 25.	(10.)	648	8	6¼	÷ 30, 64, 84.
(11.)	989	4	0¾	÷ 48, 63, 88.	(12.)	865	11	2¾	÷ 27, 49, 121.
(13.)	702	4	5½	÷ 36, 55, 108.	(14.)	897	10	1¼	÷ 45, 56, 120.
(15.)	314	17	7¼	÷ 35, 81, 132.	(16.)	991	0	10¾	÷ 72, 99, 100.
(17.)	926	5	9	÷ 77, 96, 112.	(18.)	457	16	4¼	÷ 49, 54, 144.

	£	s.	d.			£	s.	d.	
(19.)	874	15	10	÷ 37, 41, 47.	(20.)	770	11	6½	÷ 43, 53, 67.
(21.)	991	10	2¼	÷ 29, 39, 69.	(22.)	568	15	9¾	÷ 79, 87, 91.
(23.)	487	13	3½	÷ 89,97,109.	(24.)	976	0	11¾	÷ 91, 95, 97.
(25.)	5674	12	8¼	÷ 247, 351.	(26.)	6986	13	2¼	÷ 493, 569.
(27.)	7749	8	7	÷ 679, 747.	(28.)	7891	0	10¾	÷ 379, 917.
(29.)	8751	11	3½	÷ 1865,2933.	(30.)	9989	18	1¼	÷ 3891, 7468.

	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
(31.)	46	9	6	÷	3	17	5½.	(32.)	382	16	0	÷	5	19	7½.
(33.)	996	2	1¾	÷	8	4	7¾.	(34.)	989	14	6	÷	7	9	11½.
(35.)	1007	18	2	÷	12	15	2.	(36.)	1469	16	2	÷	19	6	9½.
(37.)	14046	9	5½	÷	98	4	6½.	(38.)	12502	15	2½	÷	89	18	11½.

Ex. 29.

Divide

(1.) 247 cwt. 3 qrs. 21 lbs. by 7, 33, 117.
(2.) 156 cwt. 3 qrs. 22 lbs. by 11, 18, 39.
(3.) 18 cwt. 3 qrs. 13 lbs. 8 oz. by 12, 27, 69.
(4.) 21 lbs. 4 oz. 3 dwts. 10 grs. by 10, 15, 19.
(5.) 98 wks. 1 day, 18 hrs. by 65, 79, 97.
(6.) 79 qrs. 2 bu. 1 pk. by 43, 57, 144.
(7.) 1241 tons, 14 cwt. 2 qrs. 15 lbs. by 81, 108, 287.
(8.) 1521 lbs. 10 oz. 14 dwts. 18 grs. by 471, 363, 512.
(9.) 1974 miles, 7 fur. 23 po. 5 yds. 2 ft. by 562, 642, 927.
(10.) 7124 lbs. 10 oz. 7 drs. 2 scrs. 15 grs. by 212, 256, 343.
(11.) 576 miles, 4 fur. 24 po. 4 yds. by 168, 216, 407.
(12.) 345 a. 2 r. 33 po. by 59, 87, 313.
(13.) 98 cwt. 14 lbs. by 5 qrs. 17 lbs.
(14.) 39 yds. 3 qrs. 3 nls. by 4 yds. 1 qr. 3 nls.
(15.) 2046 a. 0 r. 32 p. by 9 a. 3 r. 14 p.
(16.) 64 lbs. 6 oz. 10 dwt. by 5 dwt. 9 grs.
(17.) 1089 yds. by 15 yds. 2 nls.
(18.) 141 dys. 3 hrs. 4 min. by 17 dys. 15 hr. 23 min.
(19.) 5 bu. 2 pks. by 1 pk. 3 qts.
(20.) 4 miles, 5 fur. 179 yds. by 2 fur. 22 yds. 6 in.

Ex. 30.

Miscellaneous Examples.

(1.) If 1 cwt. 1 lb. cost £119 4s. 9¼d., what is the cost of 1 lb.

(2.) How many squares, with 9 men on each side, can be formed out of a million of men?

(3.) Divide £1901 2s. 2¼d. into 99 equal portions.

(4.) At a sale of standing corn, I bought six lots, which measured 3 a. 2 r. 19 p.; 4 a. 3 r. 26 p.; 4 a. 1 r. 33 p.; 5 a. 2 r. 23 p.; 2 a. 3 r. 19 p.; and 3 a. 2 r. respectively. Required the price at £12 per acre.

(5.) If a gentleman's income be £1265 6s. 8d. a year, how much may he spend per week and per day, so as to save one quarter of it?

(6.) How many square feet of glass are there in a window which contains 12 panes, each 18 in. by 16 in.?

(7.) How often will a cart-wheel turn, in going a distance of 40 miles, if its circumference be 3 ft. 9 in.?

(8.) The iron railings round St. Paul's weigh 200 tons 81 lbs. What did they cost, at 7d. per lb.?

(9.) A line of railway is 343 miles in length; 276 miles were completed at a cost of £393 16s. 4½d. per mile; the remainder at £433 19s. 11d. per mile; required the total expense.

(10.) A grocer bought 3 casks of sugar, each containing 3 cwt. 2 qrs. 13 lbs. The first month he sold 1 cwt. 2 qrs. 25 lbs.; the second, 2 cwt. 0 qr. 19 lbs.; and the third, 3 cwt. 1 qr. 26 lbs. How much had he taken out of the third cask?

(11.) A common field, of 286 a. 3 r. 36 p., is to be divided equally among 13 persons. How many acres will fall to the share of each?

(12.) A wheel makes 514 revolutions in passing over 1 mile 467 yds. 1 ft., what is its circumference?

(13.) A father left a small farm of 37 acres to be divided equally among his children, each of whom received 7 a. 1 r. 24 p. How many children were there?

(14.) A silversmith melted a silver tankard, weighing 5 lbs. 2 oz. 11 dwt., and made spoons of it. How many did he make, each spoon weighing 2 oz. 12½ dwt.?

(15.) Multiply £923 17s. 8½d. by 89, and divide the product by 33.

(16.) Find the price of 192 ducks at 4s. 6d. per couple, 348 pairs of shoes at 63s. 6d. per dozen, and 3582 quarters of wheat at 6s. per bushel.

17.) If a man works 5 days 8 hrs. one week, 4 days 8 hrs. the next, ıys 9 hrs. the next, and 5 days 2 hrs. the next, what will be his es for a month, if 10 hours is considered a working day, and if he aid at the rate of 6d. per hour?

18.) Six persons bought between them a piece of land, containing . 1 r. 26 p. What will be the share of each, if the land be equally ded?

19.) How many roubles, each worth 3s. 4d., are equal in value to 500 oleons at 17s. 6d. each?

20.) How many tons of coal can be purchased for £315 3s. 10¼d., 7s. 3¼d. per ton?

21.) How many cubic feet of air is there in a room which is 10 ft. . long, 8 ft. 6 in. broad, and 6 ft. 6 in. high?

22). Find the total increase of revenue for the year ending March 1865, as compared with the preceding year; and the increase or ease under each head:

Years ended	Mar. 31, 1865.	Mar. 31, 1864.
	£	£
Customs	22,572,000	23,232,000.
Excise	19,558,000	18,207,000.
Stamps	9,530,000	9,317,000.
Taxes	3,292,000	3,218,000.
Property Tax	7,958,000	9,084,000.
Post Office	4,100,000	3,810,000.
Crown Land	310,000	305,000.
Miscellaneous	2,993,436	3,035,963.

23.) Ten boxes contain each £60 15s. 6d.; find the value of the le, and express in words the number of fourpenny pieces con-ed in the amount.

24.) If £1279 13s. 8¾d. be divided into sums of £55 12s. 9¼d. h, how many such sums will there be?

25.) Add together 72 tons 5 cwt. 15 lbs., 18 tons 6 cwt. 3 qrs., 14 . 18 lbs., and multiply the sum by 56.

26.) How many days, hours, minutes, and seconds are there in 3459 seconds?

27.) In 1000 guineas how many crowns, florins, and threepenny es?

28.) If 144 tons of coals cost £181 16s.. find the price of 1 ton, and 245 tons?

(29.) How many half-crowns, shillings, and farthings, an equal number of each, are there in £169?

(30.) Find the weight of 5 dozen spoons, each weighing 2 oz. 4 dwt.

(31.) What sum added to £43 14s. 3¾d. will make 48000 farthings?

(32.) Multiply 3 tons 14 cwt. 2 qrs. 14 lbs. 12 oz. by 38, and prove the result.

(33.) How many acres, roods, poles, yards, feet, and inches are there in 40253798 square inches?

(34.) If 7½ quarters of wheat cost £23 5s., what is that per bushel?

(35.) How many half-guineas are there in a thousand half-crowns; and in 28624 inches, how many furlongs, poles, yards, feet, and inches?

(36.) A building society purchases three estates containing 35 a. 2 r. 23 p. 12½ sq. yds., 9 a. 2 r. 36 p. 27¼ sq. yds., and 40 a. 0 r. 35 p. 20¾ sq. yds. respectively. If the property be divided into 128 equal parts, of how much does each part consist?

(37.) A person laid out £23 2s. 2d. in the purchase of 163 yds. of cloth, of which 72 cost 2s. 5d. a yard; what was the price per yard of the remainder?

(38.) If eight bales of cloth, each containing 4 parcels, each parcel 10 pieces, and each piece 26 yds., cost £6656; what was the price per yard?

(39.) How many quarters of corn can be bought for 190 guineas, at 7s. 1½d. per bushel?

(40.) A bankrupt's property amounts to £455 1s. 6¾d., and his debts to £1365 4s. 8¼d.; what can he pay in the pound?

Ex. 31.

Calculate the following bills :—

(1.)	*s.*	*d.*
6 lbs. of Black Tea at...	5	0
1 lb. of Green Tea......	6	0
2 lbs. of Coffee	2	0
7 lbs. of Sugar	0	8

(2.)	*s*	*d.*
6 yds. of Linen at	3	3
4 Handkerchiefs............	5	3
2 Pairs of Gloves	3	6
1 Umbrella	18	6

(3.)	*s.*	*d.*
22 yds. of Cloth at ...	8	7
97 yds. of Dimity......	3	5
34 yds. of Carpet	5	7
79 pairs of Stockings...	2	9

(4.)	*s*	*d.*
45 yds. of Moreen at......	3	6
51 pairs of Braces	1	9
23 Silk Handkerchiefs ...	2	3
55 pairs of Shoes	3	5

(5.)

	s.	d.
111 yds. of Sheeting at	2	3
53 yds. of Lace.........	4	7
107 yds. of Silk	5	6
61 yds. of Kerseymere	11	4
113 yds. of Calico	2	7
89 yds. of Velvet......	6	1

(6.)

	s.	d
37 yds. of Velvet at	6	5
29 yds. of Irish Linen ...	4	8
43 yds. of Satin	7	2
101 yds. of Cambric.........	5	9
106 yds. of Muslin	3	7
46 yds. of Lawn	9	5

(7.)

	s.	d.
84 lbs. of Cinnamon at	7	9
96 lbs. of Pepper......	7	8½
108 lbs. of Ginger......	6	8¼
54 lbs. of Nutmegs ...	8	3½

(8).

	s.	d.
121 gallons of Rum at ...	9	5¾
99 gallons of Gin	7	8½
110 gallons of Whiskey...	9	5½
55 gallons of Brandy ...	13	9½

(9.)

	s.	d.
81 bu. of Barley at ...	6	4¼
120 bu. of Beans	7	3½
27 bu. of Wheat	12	9¾
12¼ bu. of Maize	15	7

(10.)

	s.	d.
13¾ lbs. of Congou at......	5	4
7¾ lbs. of Hyson	12	5
17½ lbs. of Gunpowder ...	11	4½
19¼ lbs. of Souchong	8	6

(11.)

	s.	d.
15½ lbs. of Allspice at	8	4½
9¼ lbs. of Nutmegs ...	5	7
15¼ lbs. of Cinnamon	9	4
16½ lbs. of Ginger......	7	8½
17½ lbs. of Cloves......	7	6½
14¾ lbs. of Pepper ...	8	1¼

(12.)

	s.	d.
19½ yds. of Velvet at	7	8½
12¼ yds. of Black Cloth...	15	7
13½ yds. of Damask	9	8½
14¾ yds. of Moreen.........	10	6
9¾ yds. of Brocade	10	6
15½ yds. of Black Silk ...	7	4

(13.)

	s.	d.
9 qrs. of Malt at ...	24	4
19 lbs. of Hops	1	3½
17 bu. of Beans	4	7
23 bu. of Peas	3	11½

(14.)

	s.	d.
15 doz. of Sherry at	43	6
17 doz. of Port	39	6
13 doz. of Stout	6	6
12 doz. of Ale	4	3

XI.—PRACTICE.

Practice is a convenient method of finding the value of any number of articles when the value of one is known.

This method consists in the right use of **Aliquot Parts.**

When a number or quantity is contained an *exact* number of times in another it is called an *aliquot part* of it.

Thus 3d. is an aliquot part of 1s., for 3d. is contained in 1s. exactly 4 times.

TABLE OF ALIQUOT PARTS.

s.	*d.*		£	*s.*	*d.*		£	*d.*		*s.*
10	0	=	$\frac{1}{2}$	2	0	=	$\frac{1}{10}$	6	=	$\frac{1}{2}$
6	8	=	$\frac{1}{3}$	1	8	=	$\frac{1}{12}$	4	=	$\frac{1}{3}$
5	0	=	$\frac{1}{4}$	1	4	=	$\frac{1}{15}$	3	=	$\frac{1}{4}$
4	0	=	$\frac{1}{5}$	1	3	=	$\frac{1}{16}$	2	=	$\frac{1}{6}$
3	4	=	$\frac{1}{6}$	1	0	=	$\frac{1}{20}$	$1\frac{1}{2}$	=	$\frac{1}{8}$
2	6	=	$\frac{1}{8}$		6	=	$\frac{1}{40}$	1	=	$\frac{1}{12}$

qrs.	lbs.		cwt.	lbs.		cwt.	lbs.		qr.
2	0	=	$\frac{1}{2}$	14	=	$\frac{1}{8}$	14	=	$\frac{1}{2}$
1	0	=	$\frac{1}{4}$	8	=	$\frac{1}{14}$	7	=	$\frac{1}{4}$
0	16	=	$\frac{1}{7}$	7	=	$\frac{1}{16}$	4	=	$\frac{1}{7}$

The above aliquot parts are given as an illustration only: many others may readily be found from the Tables of Weights and Measures.

1. *When the quantity whose value is required is a simple number.*

RULE.—Multiply the quantity or number of articles by the number of pounds, shillings, or pence of the price, and take aliquot parts for the rest. Add the results together, and their sum will be the total value.

Note 1.—When the given price is less than a pound, a shilling, or a penny, it is often most convenient to take the given number as valued in pounds, shillings, or pence, respectively, and divide by the aliquot part or parts that make up the given price.

Note 2.—The parts should be taken in order, so that each may be an *aliquot part* of some preceding part, and generally of the part *immediately* preceding.

Note 3.—It will be sometimes convenient to suppose the value *increased* so as to become an exact number of pounds or shillings, then *subtract* the excess in value.

AMPLE I.— Find the value of 746 loaves at 7½d. each.

```
      d.
     746
      7½
    ----
    5222
     373
2)5595 d.
,0) 46,6  3
  £23 6s. 3d.
```

Here 746d. is the value of 746 at 1d. each; multiplying by 7 gives the value of 746 at 7d each. Then since a ½d. is the half of a penny, the value of 746 at a ½d. each will be ½ of 746d. = 373d. This value added to the preceding gives 5595d. as the total value. Then dividing by 12 and 20 we obtain the answer £23 6s. 3d.

```
thus:  s.
= ½   746
= ¼   373
       93  3
2,0)46,6   3
   £23 6   3
```

In the second method 746s. is the value of 746 at 1s. each. Then since 6d. is ½ of 1s. the value of 746 at 6d. will be ½ of 746s. = 373s. Then since 1½d. is the ¼ of 6d. the value at 1½d. will be the fourth part of 373s. = 93s. 3d. The two values added together give 466s. 3d. or £23 6s. 3d. as before.

```
hus:  £
1/40  746
= ¼    18 13  0
        4 13  3
      £23  6  3
```

In the third method the top line is taken as the value in pounds.

Then 6d. is the 1/40 of £1; and 1½d. = ¼ of 6d.

AMPLE II.—Find the value of 5136 yards at 15s. 8½d. each.

```
6d.  | ½ |    s.
     |   |   5136
     |   |     15
     |   |  77040                   10s. | ½    £
 d.  | ⅓ |   2568                    5s. | ½   5136
½d.  | ¼ |    856        or thus     6d. | 1/10 2568
     |   |    214                    2d. | ⅓    1284
       2,0)8067,8                   ½d. | ¼     128  8  0
         4033 18  0                               42 16  0
                                                  10 14  0
                                                4033 18  0
```

AMPLE III.—What is value of 8542 ounces of gold at £3 17s. per ounce?

```
      |     | £                               £  s. d.
 10s. | ½   | 8542 value of 8542 oz. at       1  0  0
      |     |    3
      |     | 25626          ,,     ,,        3  0  0
  5s. | ½   |  4271          ,,     ,,          10  0
2s. 6d.| ½  |  2135 10 0     ,,     ,,           5  0
  3d. | 1/10|  1067 15 0     ,,     ,,           2  6
1½d.  | ½   |   106 15 6     ,,     ,,              3
      |     |    53  7 9     ,,     ,,              1½
        33260   8 3  ,,                       3 17 10½
```

Ex. 32.

Find the cost of the following articles :—

			s.	d.
(1.)	2712	at	6	8
	1575		3	4
	8646		10	0
	4838		5	0

			s.	d.
(2.)	6457	at	3	4
	5183		2	6
	9768		1	3
	4576		1	4

		s.	d.
(3.)	2954	1	3
	1463	6	8
	8241	2	6
	1958	5	0

			s.	d.
(4.)	5968	at	1	8
	9746		1	0
	7589		6	8
	5836		2	6

			s.	d.
(5.)	2378	at	1	4
	5827		1	3
	6592		1	8
	9576		3	4

		s.	d.
(6.)	9817	0	6
	9658	10	0
	6841	1	4
	3894	4	0

			s.	d.
(7.)	5827	at	6	8
	5973		5	0
	9817		10	0
	5368		2	6

		s.	d.
(8.)	5821	1	3
	5132	3	4
	4216	1	0
	1459	5	0

		s.	d.
(9.)	4163	2	6
	2568	6	8
	1649	0	6
	8720	1	4

---

			d.
(10.)	8649	at	2
	7896		3
	4987		4
	9168		6

		d.
(11.)	7358	1½
	9164	1
	4917	6
	6583	2

			d.
(12.)	8276	at	6
	5739		4
	1684		3
	6472		2

		d.
(13.)	8917	1
	6549	1½
	8572	3
	6158	2

			d.
(14.)	9648	at	4
	4719		3
	2869		6
	6814		2

		d.
(15.)	9148	1½
	5326	3
	7135	4
	9146	6

Ex. 33.

Find the cost of the following articles :—

			s	d.
(1.)	8645	at	1	2
	5837		1	2½
	9864		1	5
	8976		0	5½

		s.	d.
(2.)	5173	1	5½
	8569	0	8½
	5837	1	0½
	9168	0	7½

			s.	d.
(3.)	8592	at	0	9
	7638		0	9½
	5471		1	9
	6582		2	2

			s.	d.
(4.)	7136	at	1	6
	5847		1	6¾
	9176		0	10
	8569	.	0	10½

			s.	d.
(5.)	6845	at	1	0¾
	2976		2	3
	1084		4	6
	5736		1	1½

			s.	d.
(6)	7163	at	1	2¾
	8295		1	1½
	5617		2	6¼
	5284		3	5½

			s.	d.
(7.)	5768	at	8	3
	6215		7	4
	7463		12	2½
	9582		6	3¼

			s.	d.
(8.)	4268	at	5	8½
	5917		6	2¼
	9678		7	2½
	5716		12	5

			s.	d.
(9.)	2715	at	5	10
	5183		4	0¾
	6249		14	8
	5416		13	11

			s.	d.
(10.)	8174	at	9	6¾
	5069		18	7
	7265		6	9½
	5173		13	2

			s.	d.
(11.)	8397	at	9	11½
	2816		15	6
	4169		3	9½
	3716		17	6¾

			s.	d.
(12.)	4095	at	8	2½
	3156		16	9
	9275		7	2½
	1736		16	9½

			s.	d.
(13.)	7568	at	16	3¼
	9817		8	6½
	5689		7	0¾
	7516		12	3¾

			s.	d.
(14.)	2458	at	14	0¾
	5716		6	3½
	9847		13	9¼
	5836		8	7½

			s.	d.
(15.)	7163	at	8	7½
	9584		9	6¼
	6394		12	8¼
	7216		6	11½

			s.	d.
(16.)	9548	at	11	0¾
	1458		19	10¼
	3712		17	10¾
	9683		11	11½

			s.	d.
(17.)	2875	at	16	0¾
	8264		19	2½
	7165		16	10½
	9384		13	7¼

			s.	d.
(18.)	4876	at	19	11¼
	6259		15	11¾
	6158		16	3¾
	8394		19	7¼

2.—*When the quantity whose value is required is a mixed number, i. e., a simple whole number and a fraction.*

Rule.—Find the value of the simple number, as before, and add to it the fractional part of the rate.

The fractional parts may be found by taking ½, ¼, ⅓, &c., or by multiplying the rate by the **Numerator** (the upper figure of the fraction), and dividing the product by the **Denominator** (or lower figure).

Or thus :—Find the value of the mixed number at the unit of price, and then take, as before, the aliquot parts for the given rate.

This method is only convenient when the denominator of the fraction will *exactly* divide the unit of price.

Example I.—Find the cost of $424\frac{3}{4}$ yards, at £2 12s. 6d. per yard.

		£		
10s.	$\frac{1}{2}$	424		
		2		
		848		
2s. 6d.	$\frac{1}{4}$	212		
		53		
for $\frac{1}{2}$ yd.		1	6	3
" $\frac{1}{4}$ "			13	$1\frac{1}{2}$
		£1114	19	$4\frac{1}{2}$

		£	s.	d.
10s.	$\frac{1}{2}$	424	15	0 *
				2
		849	10	0
2s. 6d.	$\frac{1}{4}$	212	7	6
		53	1	$10\frac{1}{2}$
		£1114	19	$4\frac{1}{2}$

* Since $424\frac{3}{4}$ yards at £1 = £$424\frac{3}{4}$, that is, £424 15s. 0d.

Example II.—Find the value of $189\frac{3}{7}$ cwt. at £2 2s. $3\frac{1}{2}$d. per cwt.

		£		
2s.	$\frac{1}{10}$	189		
		2		
		378		
3d.	$\frac{1}{8}$	18	18	
$\frac{1}{2}$d.	$\frac{1}{6}$	2	7	3
			7	$10\frac{1}{2}$
for $\frac{3}{7}$ cwt.			18	$1\frac{1}{2}$
		£400	11	3

	£	s	d.	
	2	2	$3\frac{1}{2}$	× $\frac{3}{7}$
			3	
7)	6	6	$10\frac{1}{2}$	
		18	$1\frac{1}{2}$	

Ex. 34.

Find the value of the following articles:—

		s.	d.			s.	d.			£	s.	d.
(1.)	$217\frac{1}{2}$ at	7	10	(9.)	$317\frac{3}{4}$ at	16	10	(17.)	$797\frac{1}{4}$ at	1	10	7
(2.)	$896\frac{1}{4}$	9	$9\frac{1}{4}$	(10.)	$801\frac{3}{8}$	17	4	(18.)	$853\frac{3}{4}$	8	14	9
(3.)	$547\frac{2}{3}$	8	$7\frac{1}{2}$	(11.)	$417\frac{2}{11}$	18	$1\frac{1}{4}$	(19.)	$901\frac{1}{2}$	5	11	$7\frac{1}{2}$
(4)	$389\frac{1}{5}$	17	$2\frac{1}{4}$	(12.)	$628\frac{3}{4}$	14	8	(20.)	$796\frac{3}{5}$	2	18	$0\frac{1}{4}$
(5.)	$764\frac{5}{6}$	12	$10\frac{1}{2}$	(13.)	$723\frac{2}{9}$	19	$3\frac{3}{4}$	(21.)	$853\frac{7}{8}$	1	13	2
(6.)	$593\frac{1}{8}$	19	2	(14.)	$514\frac{6}{7}$	15	$5\frac{1}{2}$	(22.)	$365\frac{5}{6}$	2	12	$7\frac{1}{2}$
(7.)	$761\frac{2}{7}$	13	$6\frac{3}{4}$	(15.)	$491\frac{4}{5}$	17	$7\frac{1}{4}$	(23.)	$219\frac{5}{11}$	5	16	$6\frac{1}{2}$
(8.)	$476\frac{1}{9}$	18	$11\frac{1}{4}$	(16.)	$687\frac{5}{8}$	17	2	(24.)	$276\frac{3}{8}$	2	12	$6\frac{3}{4}$

3.—*When both the quantity and the rate are of several denominations.*

Rule.—Multiply the price or rate by the number of the same denomination as the unit whose value is given, and take fractional parts of the price for the lower denominations. Add the results together and their sum will be the total value.

EXAMPLE I.—Find the value of 5 cwt. 3 qrs. 7 lbs. at £12 13s. 4d. per cwt.

		£	s.	d.		cwt.	qrs.	lbs.
2 qrs.	½	12	13	4 =	value of	1	0	0
				5				
		63	6	8 =	,, ,,	5	0	0
1 qr.	½	6	6	8 =	,, ,,		2	0
7 lbs.	¼	3	3	4 =	,, ,,		1	0
			15	10 =	,, ,,			7
		£73	12	6 =	,, ,,	5	3	7

EXAMPLE II.—What will be the cost of 73 acres, 3 roods, 12 poles, at £33 13s. 4d. per acre?

		£	s.	d.		a.	r.	p.
2 r.	½	33	13	4	cost of	1	0	0
				9				
		303	0	0		9	0	0
				8				
		2424	0	0	,, ,,	72	0	0
		33	13	4	,, ,,	1	0	0
1 r.	½	16	16	8	,, ,,		2	0
8 p.	⅕	8	8	4	,, ,,		1	0
4 p	½	1	13	8	,, ,,			8
			16	10	,, ,,			4
		2485	8	10	,, ,,	73	3	12

Ex. 35.

Find the value of the following:—

(1.) 84 cwt. 3 qrs. 14 lbs. of sugar, at £6 5s. 10d. per cwt.
(2.) 37 cwt. 2 qrs. 14 lbs., at £7 10s. 9d. per cwt.
(3.) 25 cwt. 3 qrs. 14 lbs. 10 oz., at £5 10s. 6d. per cwt.
(4.) 73 cwt. 2 qrs. 7 lbs., at £12 3s. 4d. per cwt.
(5.) 144 cwt. 3 qrs. 21 lbs., at £4 2s. 6d. per cwt.
(6.) 87 cwt. 3 qrs. 16 lbs., at £5 5s. 9d. per cwt.
(7.) 79 cwt. 0 qr. 13 lbs., at £3 11s. 2¼d. per cwt.
(8.) 75 cwt. 2 qrs. 23 lbs., at £4 17s. 9d. per cwt.
(9.) 197 cwt. 3 qrs. 11 lbs., at £4 17s. 7¾d. per cwt.
(10.) 336 cwt. 2 qrs. 17 lbs., at £7 15s. 6¾d. per cwt.
(11.) 372 cwt. 3 qrs. 9 lbs., at £4 13s. 9¾d. per cwt.
(12.) 371 cwt. 2 qrs. 19 lbs., at £3 1s. 4d. per cwt.
(13.) 297 cwt. 3 qrs. 6 lbs., at £2 7s. 11¾d. per cwt.
(14.) 417 cwt. 0 qr. 27 lbs., at £3 3s. per cwt.
(15.) 327 cwt. 3 qrs. 22 lbs., at £3 13s. 2½d. per cwt.

(16.) 137 cwt. 3 qrs. 23 lbs., at £4 13s. 7½d. per cwt.
(17.) 89 cwt. 3 qrs. 27 lbs., at £2 6s. 8d. per cwt.
(18.) 65 cwt. 3 qrs. 25 lbs., at 112s. per cwt.
(19.) 30 ac. 1 r. 4 p., at £3 10s. per acre.
(20.) 80 ac. 3 r. 24 p., at £2 3s. 4d. per acre.
(21.) 98 ac. 1 r. 18 p., at £5 19s. 10d. per acre.
(22.) 57 ac. 3 r. 26 p., at £14 2s. per acre.
(23.) 127 ac. 3 r. 37 p., at £3 6s. 8d. per acre.
(24.) 152 ac. 2 r. 15 p., at £3 15s. per acre.
(25.) 197 sq. yds. 8 sq. ft. 80 sq. in., at £3 13s. 7¾d. per square yard.
(26.) 28 sq. yds. 3 sq. ft. 48 sq. in., at £1 13s. 4d. per square yard.
(27.) 91 cub. yds. 13½ cub. ft., at £1 16s. 10d. per cubic yard.
(28.) 22 oz. 16 dwts. 8 grs. at £3 4s. 6d. per oz.
(29.) 144 oz. 16 dwts. 18 grs., at £1 15s. per oz.
(30.) 31 oz. 15 dwts. 13 grs., at 7s. 7½d. per oz.
(31.) 86 lbs. 3 oz. 15 dwts. 18 grs., at £4 16s. 4d. per *oz.*
(32.) 15 yds. 2 ft. 8 in., at £1 19s. per yard.
(33.) 38 yds. 1 ft. 7 in., at £2 3s. 7½d. per yard.
(34.) 17 yds. 2 ft. 10 in., at 13s. 3½d. per *foot.*
(35.) 6 mo. 2 w. 4 d., at £1 11s. 6d. per month.
(36.) 7 mo. 3 w. 6 d., at £3 3s. per *week.*
(37.) 637 qrs. 4 bush. 2 pks., at £3 12s. per quarter.
(38.) 124 qrs. 4 bush. 4 gal., at £2 5s. per quarter.
(39.) 18 gal. 2 qts. 1 pt., at £1 12s. 8d. per gallon.
(40.) 95 bush. 3 pks. 1 gal., at 4s. 4d. per bushel.

Ex. 36.

(1.) What is the income tax on £179 17s. 6d., at sixpence in the pound?

(2.) If an estate be worth £384 16s. a year, and the land-tax be assessed at 2s. 9½d. per pound, what is the net annual income?

(3.) What will be the cost of relieving 165,000 poor persons, at 1s. 3½d. per head?

(4.) Find the value of 21 pockets of hops, each weighing 1 cwt. 2 qrs. 6 lbs., at £5 15s. 6d. per cwt.

(5.) Find the rent of a farm of 225 a. 1 r. 19 p, at £2 12s. 10d. per acre.

(6.) Add together 75 tons 3 cwt. 2 qrs. 12 lbs., 33 tons 1 cwt. 16 lbs., and 125 tons 2 cwt. 3 qrs. 27 lbs. and find the value of the whole at 5s. 7¾d. per *quarter.*

(7.) Find the expense of lodgings for 3 mo. 2 wks. 5 days, at a guinea per week.

(8.) What is the value of 27 lbs. 4 oz. 10 dwts. of gold dust, at £3 10s. 10½d. per *ounce*, and of 360 oz. of silver, at 63¾d. per ounce?

(9.) How much pure gold is there in 54 lbs. 9 oz. of coin, allowing 1 oz. 4 dwt. of alloy to each pound?

(10.) Find the costs of trenching 27 ac. 1 r. 32 p., at 2½d. per square yard.

(11.) What will be the cost of paving 5 miles 3 fur. 200 yds. of road, at £135 10s. 6d. per mile?

(12.) If a fence cost 2s. 8d. per yard, how much will it cost to enclose a field 427 yds. long, and 128 wide?

(13.) If 2 cwt. 3 qrs. of sugar be bought at the rate of 4d. per pound, and sold for £5 15s. 6d., how much is gained?

(14.) A bankrupt's debts amount to £869 10s., and he can pay only 11s. 6d. in the pound. What is his estate worth? and how much will a creditor, to whom he owes £57 6s. 8d., receive?

(15.) A grocer mixes 2 cwt. 0 qr. 15 lbs of sugar, at £5 2s. 8d. per cwt., with 4 cwt. 1 qr. 2 lbs., at £2 6s. 8d. per cwt. If he sells the mixture at £3 5s. 4d. per cwt., what will he gain or lose by the transaction?

(16.) 22 cwt. of tea were bought in China for £28 14s. 6d. The freight was £3 1s. 4d. per cwt.; the duty 1s. 3d. per lb.; the wharfage and other expenses, £23 7s. 6d. The whole was sold at 3s. 4d. per lb. What was gained or lost?

(17.) What will be the cost of replacing a cistern, to weigh 4 cwt. 2 qrs. 7 lbs., at the rate of £2 6s. 4d. per cwt., if the plumber allows £1 10s. 4d. per cwt. for the lead of the old one, which weighs 3 cwt. 1 qr. 19 lbs.?

(18.) An estate of 134 ac. 3 r. 16 p. is rented at £2 12s. 6d. per acre, and the best pasture, consisting of 51 ac. 2 r. 12 p., being let off at £3 10s. per acre, what has the first tenant still to pay to make up the rent?

(19.) Make out a bill for the following articles:—

156 yds. 1 qr. 3 n., at 6s. 3d. per yard; 48 yds. 1 qr. 1 n., at 1s. 5½d. per yard; and 117 yds. 1 qr. 1 n., at 5s. 1½d. per yard.

(20.) 274 yds. of linen, at 3s. 5½d.; 239 yds., at 2s. 11d.; 179 yds. of Irish, at 2s. 3½d.; 175, at 1s. 9¾d.

(21.) Find the weight of 247 bush. 5 gal. of wheat, at 62 lbs. per bushel.

(22.) What is the duty on 188 cwt. 1 qr. 21 lbs. of sugar, at 30s per cwt

(23.) What is the freight on 61 cwt. 3 qrs. 10 lbs. of coffee, at 10s. 6d. per cwt.

(24.) Find the freight on 35 cwt. 1 qr. 17 lbs. of cotton, at 1½d. per lb.

(25.) What is the yearly rent of 547 acres of land, at 17s. 6d. the acre.

(26.) What is the income tax on a rent of £700, at the rate of 7d. in the pound?

(27.) What will 2 cwt. 3 qrs. 4 lbs. of raisins come to, at £3 10s. 6d. per cwt.?

(28.) What is the price of a gold snuff-box, weighing 10 oz. 10 dwts., at £4 3s. 4d. per oz.?

(29.) Find the price of 8 cwt. 1 qr. 12 lbs., at £4 12s. 5½d. per quarter.

(30.) Find the value of 373 cwt., at £2 16s. 10¼d. per cwt.

(31.) What is the amount of tax upon £1537 10s., at the rate of 3s. 6d. per pound?

(32.) Find the price of 9 tons 11 cwt. 2 qrs. 21 lbs., at £10 11s. 4d. per cwt.

(33.) What is the price of 17 ac. 3 r. 15 p., at £60 3s. 4d. per acre?

(34.) Find the value of 9 tons 4 cwt. 3 qrs. 21 lbs., at £14 15s. 9d. per ton?

(35.) What is the dividend on £2734 16s. 8d., at 9s. 4½d. in the pound?

(36.) Find the price of 217¼ yards of lace, at £2 17s. 7½d. per yard.

(37.) Find the price of 104 ac. 3 r. 10 p., at £42 15s. an acre.

(38.) What is the price of 329 yds. 3 qrs. 2 nls., at 5s. 2½d. per quarter?

(39.) Find the price of 60 lbs. 8 oz. of gold, at £3 10s. 11¼d. per oz.

(40.) Bought 4 pockets of hops, each weighing 1 cwt. 1 qr. 21 lbs., at £4 2s. 6d. per cwt. What is the cost?

(41.) Soap at £3 11s. 6d. the cwt., what is the value of 53 cwt. 17 lbs.?

(42.) What is the cost of 3 bags of rice, each containing 182 lbs., at 38s. 4d. per cwt.?

(43.) If 30 yards of cloth, worth 5 guineas the dozen yards, are sold, and the proceeds divided between 20 persons, what is the share of each?

(44.) What is the rent of 67 acres of land for 2 years 9 months 11 days, the yearly rent of an acre being £2 15s. 6d.

(45.) If a person's estate be worth 3000 guineas a-year, and the land-tax be assessed at 2s. 9½d. in the pound, what is his net annual income?

Ex. 37

Calculate the following bills, and write them out in proper form, with names and dates:—

(1.) London, May 24th, 1865.

Messrs. A. and B.

Bought of SMITH and TAYLOR.

	£	s.	d.
20 Ps. Linen, each 25 yds., at 3s. 0d.			
16 ,, ,, ,, 26 ,, ,, 4s. 9d.			
14 ,, Merino, ,, 32 ,, ,, 5s. 3d.			
19 ,, Cotton, ,, 75 ,, ,, 0s. 5½d.			
£			

(2.) 15 pieces Muslin, each 24 yds., at 13d.; 14 pieces, each 19 yds., at 2s. 4d.; 12 pieces blue Gingham, each 28 yds., at 14d.; 30 pieces pink ditto., each 30 yds., at 1s. 8d.

(3.) 127½ lbs. Tea, at 3s. 9d. per lb.; 133 lbs. Sugar, at 4¾d.; 9 sugar-loaves, each 14¼ lbs., at 8¼d.; 188¼ lbs. Coffee, at 1s. 4½d.; 87¾ lbs. Prunes, at 1s. 1¾d.

(4.) Coffee, No. I., 370¼ lbs., at 1s. 1d.; No. II., 470¾ lbs., at 1s. 2d.; No. III., 511 lbs., at 1s. 4d.; No. IV., 486½ lbs., at 1s. 5d.

(5.) 869 lbs. Hyson, at 4s. 7d.; 762 lbs. Congou, at 3s. 6d.; 513 lbs Bohea, at 2s. 1½d.; 115 lbs. Raisins, at 6½d.

(6.) 176 reams Demy, at 14s.; 220 reams Royal, at 17s. 4d.; 110 reams Small Post, at 14s. 9d.

(7.) 476 qrs, 3 bush. Wheat, at 41s. 5d. per qr.; 326 qrs. 4 bush. Barley, at 38s. 7d.; 127 qrs. 5 bush. Oats, at 23s. 3d.; 146 qrs. 7 bush. Beans, at 39s. 11d.

(8.) 521 galls. Old Rum, at 8s. 3½d. per gal.; 217 galls. Brandy, at 7s. 11¼d.; 192½ galls. Gin, at 4s. 9½d.; 137 dozen Sherry, at 36s. 6d. per dozen; 158 dozen Port, at 57s. 6d.

(9.) 40 pieces of Calico, each 30 yds., at 2s. 4d. per yard; 21 pieces, each 27½ yds., at 3s. 8d.; 25 pieces, each 26 yds., at 2s. 7½d.; 16 pieces, each 25 yds., at 2s. 2½d.; 17 pieces, each 29 yds., at 2s. 3¼d.; 55 yards, at 4s. 2d.

(10.) 13 cwt 1 qr. 14 lbs. Ginger, at £4 13s. per cwt.; 7 cwt. 2 qrs 27 lbs. Pimento, at £3 18s.; 3 hhds. Log-wood, each 4 cwt. 3 qrs. 27 lbs., at 14s. 6d.; 115 cwt. 1 qr. 19 lbs. Alum, at 18s. 4d.

(11.) Plain Linen, 550 yds., at 2s. 8¾d. per yard; striped ditto, 654 yds., at 2s. 8⅓d.; flowered ditto, 780 yds., at 13s. 4½d.; sprigged ditto, 660 yds., at 7s. 8¾d.; cambrics, 950 yds., at 7s. 8⅛d.; striped ditto, 460 yds., at 13s. 4¼d.; flowered ditto, 468 yds., at 7s. 8½d.; silk gauzes 860 yds., at 3s. 11¼d.

(12.) 174⅝ lbs. Quinquina, at £3 14s. 8d. per lb.; 321⅗ lbs. Gum Lac, at 5s. 9d.; 607⅛ lbs. Rhubarb, at 12s. 4d.; 720½ lbs. Mastich, at 1s. 0¾d.; 509⅕ lbs. Sasafras, at 0s. 6½d.

XII.—ALLOWANCES ON GOODS.

Certain allowances are made on some kinds of goods for the weight of the boxes, chests, casks, &c., containing them.

The whole weight of the goods and package is called the **Gross Weight.**

The allowance made for the weight of the package is called **Tare,** and is either so much in the whole, so much per package, or so much per cwt.

Another small allowance, called **Draft,** is sometimes made to retail dealers, on account of what is termed "the turn of the scale."

The result, after *all* the allowances have been subtracted, is called the **Nett Weight.**

The amount of these allowances can always be calculated by Multiplication or Practice.

The allowances called **Trett** and **Cloff** are not used at the present time

MPLE I.—What is the nett weight of 6 hhds. of sugar, each g 9 cwt. 3 qrs. 17 lbs. gross; tare, 1 cwt. 2 qrs. 2 lbs. per hhd.?

cwt.	qrs.	lbs.	
9	3	17	**Gross** weight of 1 hhd
1	2	2	**Tare** on „ „
8	1	15	**Nett** weight of „ „
		6	
50	1	6	**Nett** weight of 6 hhds.

IPLE II.—Find the nett weight of the following 4 tierces of coffee.

	cwt.	qrs.	lbs			cwt.	qrs.	lbs.
Gross,	7	2	4		**Tare,**	0	3	1
„	5	3	4		„	0	2	20
„	6	2	20		„	0	2	27
„	7	1	3		„	0	3	3
	27	1	3			2	3	23
	2	3	23	**Tare.**				
	24	1	8	**Nett** weight.				

MPLE III.—Find the nett weight of 10 casks of madder, the gross weight of each cask being 13 cwt. 3 qrs. 10 lbs.; draft, er cask; tare, 12 lbs. per cwt.

		cwt.	qrs.	lbs.	
		13	3	10	**Gross** weight of 1 cask.
				5	**Draft** allowed on „ „
		13	3	5	**Draft** suttle* of „ „ .
				10	
8 lbs.	$\frac{1}{14}$	137	3	22	„ „ „ 10 casks.
4 lbs.	$\frac{1}{2}$	9	3	11	**Tare** at 8 lbs. per cwt.
		4	3	19	„ „ 4 lbs. „ „
		14	3	2	„ „ 12 lbs. „ „
		123	0	20	**Nett** weight.

---

hen only a part of the allowances has been subtracted, the remainder times called *Suttle.*

Ex. 38.

Find the nett weight of the following:—

(1.) Six bags of Cotton, each 3 cwt. 1 qr. 18 lbs.; *tare* 5½ lbs. per bag.

(2.) 66 cwt. 1 qr. 8 lbs. of Sugar; *tare* 14 lbs. per cwt.

(3.) Four hogsheads of Sugar; No. I., 14 cwt. 1 qr. 16 lbs.; No. II., 13 cwt. 3 qrs. 11 lbs.; No. III., 15 cwt. 1 qr. 19 lbs.; No. IV, 16 cwt. 1 qr. 8 lbs.: *tare* 16 lbs. per cwt.; *draft* 2 lbs. per 3 cwt.

(4.) Five chests Indigo, each 2 cwt. 1 qr. 17 lbs.; *tare* 13 lbs. per cwt.

(5.) Seven hogsheads of Sugar, each weighing 10 cwt. 3 qrs. 7 lbs. gross; *tare* 1 cwt. 1 qr. 12 lbs. per hogshead?

(6.) Five bales of East India cotton; viz. 3 cwt. 0 qr. 23 lbs.; 3 cwt. 1 qr. 11 lbs.; 2 cwt. 3 qrs. 26 lbs; 3 cwt. 1 qr. 14 lbs.; 2 cwt. 3 qrs. 20 lbs.; allowing 8 lbs. on each bale for ropes, 1 lb. per bale for *draft*, and *tare* 4 lbs. per cwt.

(7.) What will 50 bags of coffee come to, at 14½d. per lb., each bag containing 17½ lbs. of coffee; *tare* 2½ lbs. per bag?

(8.) Nine hhds. sugar, each 3 cwt. 2 qrs. 21 lbs., *tare* 12½ lbs. per cwt., at £3 12s. 7¼d per cwt.

(9.) Find the value of 10 bales East India cotton, weighing in the gross 62 cwt. 1 qr. 23 lbs., allowing 8 lbs. per bale for ropes, 1 lb. per bale for *draft*, and *tare* 4 lbs. per cwt., at 5¼d. per lb.

(10.) Bought 10 bales of Bourbon cotton, weighing 25 cwt. 2 qrs. 6 lbs.; *draft* 1 lb. per bale; and *tare* 4 lbs. per cwt. What will the nett weight cost, at 1s. 2d. per lb.?

(11.) What will be the value of 20 bales of American cotton, weighing 73 cwt. 0 qr. 16 lbs., at 5⅛d. per lb., allowing 1 lb. on each bale for *draft*, and 4 lbs. per cwt. for *tare?*

(12.) What is the value, at 1s. 4d. per lb. nett., of 90 bales cotton wool, weighing gross 135 cwt. 0 qr. 23 lbs.; *draft* 1 lb. per bale, and *tare* 2½ lbs. per 100 lbs.?

(13.) How much will 7 casks of oil, each weighing gross 2 cwt. 2 qrs. 15 lbs., *tare* 30 lbs. per cask, cost, at £5 5s. per cwt.?

(14.) Find the value of 10 casks of butter, each weighing 4 cwt. 0 qr. 6 lbs., *tare* 18 lbs. per cwt., at £5 10s. 6d. per cwt.

(15.) Find the value of 13 butts of currants, each weighing 7 cwt. 1 qr. 10 lbs., *tare* 14 lbs. per cwt., at £6 12s. 8d. per cwt.

XIII.—MEASURES AND MULTIPLES.

One number is said to be a **Measure** or **Factor** of another, when it *can divide it exactly* without any remainder (pages 13 and 21).

A number which can divide *two or more* numbers without any remainder, is called a **Common Measure**, or **Common Factor**, of those numbers.

The greatest number which can so divide them is, of course, their **Greatest Common Measure—G.C.M.**

Thus, 2, 3, and 6, are each common measures of 18, 30, and 48; but 6 is the G.C.M., being the greatest number that can divide each of them without a remainder.

A number that has no factors, except itself and unity,* is called a **Prime Number**; as, 1, 2, 3, 5, 7, 11, &c.

When two or more numbers have no common measure or factor, except unity, they are said to be *prime to each other.*

Thus, 13 and 27 are prime to each other, because they have no common measure; whereas 13 and 26 are *not* prime to each other, because they are each divisible by 13.

No *even* number besides 2 is a prime number, for every even number is divisible by 2.

It will be well also to notice here that a number is always divisible:—

By 2, when the last figure is divisible by 2.

By 4, when the last two figures are divisible by 4.

By 8, when the last three figures are divisible by 8.

By 5, when the last figure is 5 or 0.

By 3, when the sum of the figures is divisible by 3.

By 9, when the sum of the figures is divisible by 9.

By 11, when the difference between the sum of the figures in the odd and even places is 0, or a number divisible by 11.

---

* Unity is not, however, generally named as a measure or factor of a number.

To resolve a number into prime factors, that is, factors which are prime numbers.

Rule.—Divide the number by whatever prime number will divide it exactly: divide the quotient in the same manner; and continue the process until the quotient itself is a prime number: the divisors and remaining quotient will be the prime factors.

Example.—Resolve 2275 and 4410 into prime factors.

5)2275
5)455
7)91
13

$\therefore 2275 = 5 \times 5 \times 7 \times 13.$

2)4410
5)2205
3)441
7)147
3)21
7

$\therefore 4410 = 2 \times 3 \times 3 \times 5 \times 7 \times 7.$

Ex. 39.

(1.) Write down the prime numbers from 1 to 100.
(2.) What prime numbers lie between 200 and 300?

Resolve the following into prime factors:—

(3.) 75.
(4.) 98.
(5.) 147.
(6.) 125.
(7.) 135.
(8.) 168.
(9.) 196.
(10.) 231.
(11.) 385.
(12.) 624.
(13.) 945.
(14.) 1188.
(15.) 1296.
(16.) 1331.
(17.) 1617.
(18.) 2310.
(19.) 5575.
(20.) 6171.
(21.) 8008.
(22.) 15120.

GREATEST COMMON MEASURE.

1.—*To find the Greatest Common Measure,* G.C.M., *of two numbers.*

Rule.—Divide the greater by the less; then, if there be a remainder, bring down the former divisor, and divide it *by the remainder,* and so on continually, till there be no remainder; the *last divisor* is the Greatest Common Measure.

If the last divisor, by this process, be 1, the numbers have no common measure, and are prime to each other.

2.—*To find the Greatest Common Measure*, G.C.M., *of three or more numbers.*

RULE.—Find the **G.C.M.** of any two of them; then the **G.C.M.** of the number so found and another of the given numbers; and so on to the last.

The **G.C.M.** may also be obtained by finding the *prime factors* of each number, and multiplying the common factors together.

After a little practice, the **G.C.M.** may readily be found by inspection, when the numbers are small.

EXAMPLE I.—Required the **G.C.M.** of 378 and 770.

By Division.

```
378)770(2
    756
    ---
     14)378(27
        28
        --
         98
         98
         --
         ..
```

G.C.M. = 14.

By Factors.

378 = 2 × 3 × 7 × 9.

770 = 2 × 5 × 7 × 11.

The common factors are 2 and 7;

∴ **G.C.M.** = 2 × 7 = 14.

EXAMPLE II.—Find the **G.C.M.** of 198, 495, and 660.

```
198)495(2
    396
    ---
     99)198(2
        198
        ---
        ...
```

```
99)660(6
   594
   ---
    66)99(1
       66
       --
       33)66(2
          66
          --
          ..
```

G.C.M. = 33.

Or thus: 198 = 2 × 3 × 3 × 11.

495 = 3 × 3 × 5 × 11.

660 = 2 × 2 × 3 × 5 × 11.

The factors common to all three numbers are 3 and 11; ∴ the **G.C.M.** = 3 × 11 = 33.

Ex. 40.

Find the Greatest Common Measure of

(1.) 18, 45.
(2.) 20, 84.
(3.) 57, 63.
(4.) 63, 99.
(5.) 66, 121.
(6.) 128, 162.
(7.) 176, 231.
(8.) 272, 328.
(9.) 217, 343.
(10.) 774, 819.
(11.) 330, 10021.
(12.) 1164, 1261.

(13.) 7056, 11088.
(14) 10395, 16819.
(15.) 4656, 6488.
(16.) 6330, 42360.
(17.) 43902, 49593.
(18.) 28845, 82719.
(19.) 571428, 666666
(20.) 30599, 271469.

(21.) 4802, 5145, 7546.
(22.) 2904, 3696, 9856.
(23.) 2308, 7501, 8655.
(24.) 3944, 6409, 7395.
(25.) 2262, 5655, 6032.
(26.) 5040, 5292, 4560.

(27.) Two war vessels, having the same number of guns, fire a number of broadsides. One has fired 588 shots, and the other 378. How many guns has each ship, and how many broadsides has each fired?

(28.) What is the largest sum of money contained exactly in £35 6s. 6d. and £37 10s. 6d.?

(29.) A gentleman distributed £4 18s. 2d. among a number of boys, and £2 3s. 11d. among a number of girls, giving to each child the same amount. How much could each receive, without leaving any remainder? and how many boys and girls were there?

(30.) Three regiments, containing respectively 540, 648, and 810 men, were divided into companies of equal strength. How many men will there be in a company, so that the number of companies may be as small as possible?

LEAST COMMON MULTIPLE.

1.—*To find the Least Common Multiple,* L.C.M., *of two numbers.*

Rule.—Divide the product of the two numbers by their G.C.M.: the quotient will be their Least Common Multiple.

Or thus: Divide one of the numbers by their G.C.M., and multiply the quotient by the other number.

Example.—Find the L.C.M. of 196 and 343.

G.C.M. of 196 and 343 = 49.

$$\therefore \text{L.C.M.} = \frac{\overset{4}{\cancel{196}} \times 343}{\cancel{49}} = 1372.$$

2.—*To find the* L.C.M. *of three or more numbers.*

Rule.—Find the L.C.M. of two of the numbers, then the L.C.M. of that multiple and one of the remaining numbers; and so on. The last common multiple will be the L.C.M. required.

Example.—Required the L.C.M. of 108, 207, and 216.

$$\left.\begin{aligned} 108 &= 9 \times 12 \\ 207 &= 9 \times 23 \end{aligned}\right\} \therefore \text{G.C.M.} = 9.$$

$$\text{L.C.M. of 108 and 207} = \frac{\overset{12}{\cancel{108}} \times 207}{\cancel{9}} = 2484.$$

$$\text{Again, } \left.\begin{aligned} 2484 &= 9 \times 12 \times 23 \\ 216 &= 9 \times 12 \times 2 \end{aligned}\right\} \therefore \text{G.C.M.} = 9 \times 12 = 108.$$

$$\text{L.C.M. of 2484 and 216} = \frac{2484 \times \overset{2}{\cancel{216}}}{\cancel{108}} = 4968.$$

$\therefore$ 4968 is the L.C.M. required.

When the given numbers are prime to each other, their continued product is their L.C.M.

Thus: L.C.M. of 3, 5, 7, 11, is 3 × 5 × 7 × 11 = 1155.

3.—*The following is the most convenient way of finding the* L.C.M. *of several numbers:*—

Rule.—Write the numbers in a line; strike out any number that is *exactly* contained in another.*

Divide the remaining numbers by any *prime* number that will exactly divide two or more of them: set down the quotients under the numbers so divided, and bring do the undivided ones unaltered.

---

* For any number that is a measure of another is a measure of all the *multiples of that other.*

Proceed in a similar way with the line thus formed, an repeat the process until a line is obtained, containing onl numbers which are prime to each other.

The continued product of all the divisors and the numbers in the last line will be the L.C.M. required.

Example.—Find the L.C.M. of 21, 28, 32, 36, 42, 56, and 63.

$$\begin{array}{l}
2)\underline{\cancel{21},\ \cancel{28},\ 32,\ 36,\ 42,\ 56,\ 63}\\
2)\underline{\qquad 16,\ 18,\ \cancel{21},\ 28,\ 63}\\
2)\underline{\qquad 8,\ \ 9,\ \qquad 14,\ 63}\\
\qquad\quad 4,\ \qquad\quad \cancel{7},\ 63
\end{array}$$

$$\text{L.C.M.} = 2 \times 2 \times 2 \times 4 \times 63 = 2016.$$

N.B.—It is advisable to begin the division with the *lowest prime* number, and continue dividing by it as often as possible, since the result frequently varies according to the arrangement of the divisors.

Thus, the previous example will give two different results, if the divisors are taken differently,

$$\begin{array}{l}
8)\underline{32,\ 36,\ 42,\ 56,\ 63}\\
9)\underline{\ \cancel{4},\ 36,\ 42,\ \ \cancel{7}.\ 63}\\
2)\underline{\qquad 4,\ 42,\qquad \cancel{7}}\\
\qquad\quad 2,\ 21
\end{array}
\qquad
\begin{array}{l}
6)\underline{32,\ 36,\ 42,\ 56,\ 63}\\
8)\underline{32,\ \ 6,\ \ \cancel{7},\ 56,\ 63}\\
2)\underline{\ 4,\ \ 6,\qquad \cancel{7},\ 63}\\
\quad\ \ 2,\ \ \cancel{3},\qquad\quad 63
\end{array}$$

$$8 \times 9 \times 2 \times 2 \times 21 = 6048. \qquad 6 \times 8 \times 2 \times 2 \times 63 = 12096.$$

the former result being 3 times, and the latter 6 times, too great.

Ex. 41.

Find the Least Common Multiple of

(1.) 15, 21, 45, 63.
(2.) 24, 30, 32, 48.
(3.) 54, 63, 81, 96.
(4.) 21, 44, 132.
(5.) 35, 100, 175.
(6.) 108, 144, 240.
(7.) 63, 72, 198.
(8.) 7568, 9504.
(9.) 5415, 43320.
(10.) 183, 427, 488.
(11.) 224, 343, 496.
(12.) 249, 581, 923.
(13.) 55, 45, 40, 66, 84.
(14.) 16, 42, 84, 144, 216.
15.) 255, 289, 1023, 341.
(16.) 207, 324, 567, 960.
17.) 315, 567, 693, 498.
(18.) 1023, 1445, 4095.
9.) 1575, 1800, 2760.
(20.) 5016, 2541, 3762.

(21.) Find the smallest number divisible by each of the nine digits.

(22.) Find the Least Common Multiple of the first ten even numbers.

(23.) Find the L.C.M. of the first eight odd numbers.

(24.) Find the smallest sum that could be paid with either half-crowns, florins, sixpences, or four-penny pieces.

(25.) A number of coins can be arranged in piles of 4, 6, or 9 each, without any remaining; but when arranged in piles of 7 each, there are 4 remaining. What is the smallest number of coins with which this is possible?

XIV.—VULGAR FRACTIONS.

A FRACTION is a part of a unit or any number of units.

In Vulgar Fractions, as in the arithmetic of concrete quantities, both the *number* and *denomination* of the things under consideration must be attended to. When pounds, hundredweight, ounces, &c., were the things to be considered, the symbols £, cwt., oz., &c., were written against the given *number*, to name what *kind* of things was meant. So in Vulgar Fractions, *both* number and denomination must be expressed, the *number* of things beirg written *above*, and the kind, or *name* of the things, *below*, a short line of separation.

A Simple Fraction is, therefore, expressed by two numbers placed one over the other, with a line between them.

The lower number is called the **Denominator**, and shews into how many equal parts the unit is divided.

The upper number is called the **Numerator**, and tells how many of those equal parts are to be taken to form the fraction.

Thus, in the fraction $\frac{5}{7}$, *five-sevenths*, the denominator 7 shows that the unit is to be divided into 7 equal parts; while the numerator 5 shows that 5 of those parts are under consideration.

The notation for fractions is the same as the notation for division; so that $\frac{5}{7}$ not only means *five-sevenths* of one unit, or 1, but also 5 divided by 7, or one-seventh of 5 units.

If the unit were a guinea, or 21s., $\frac{5}{7}$ of a guinea would be equal to five times one-seventh of a guinea = 5 times 3s. = 15s., and this is evidently the same as $\frac{1}{7}$ of 5 guineas, for 5 guin. = 105s., and 105s. ÷ 7 = 15s.

A **Proper** fraction is one whose numerator is less than its denominator, as $\frac{4}{5}$, $\frac{6}{11}$, $\frac{99}{119}$.

An **Improper** fraction is one whose numerator is greater than its denominator, as $\frac{16}{4}$, $\frac{15}{8}$, $\frac{25}{6}$, &c.

A **Mixed** number is made up of a whole number and a fraction, as $4\frac{1}{5}$, $7\frac{1}{8}$.

A **Compound** fraction is a fraction of a fraction, as $\frac{7}{8}$ of $\frac{4}{5}$, $5\frac{2}{3}$ of $4\frac{1}{3}$, $\frac{1}{2}$ of $7\frac{3}{4}$ of $6\frac{5}{8}$ of $\frac{11}{15}$.

A **Complex** fraction is one of which either the numerator, or denominator, or both, are either fractions, or mixed numbers, as $\dfrac{\frac{1}{14}}{\frac{3}{17}}$, $\dfrac{\frac{8}{7}}{11}$, $\dfrac{9}{2\frac{6}{13}}$, $\dfrac{2\frac{7}{9}}{3\frac{4}{5}}$.

Every whole number may be written as a fraction with a denominator 1; thus, 12 is $\frac{12}{1}$; 99 is $\frac{99}{1}$.

Unity, or 1, may be expressed by any fraction whose numerator and denominator are equal; thus, $1 = \frac{7}{7}$, $\frac{12}{12}$, $\frac{49}{49}$, $\frac{112}{112}$, &c.; for since all the parts into which the unit is divided are taken, the unit or whole is taken.

1.—*To multiply a fraction by a whole number.*

RULE.—Multiply the numerator by the whole number, or, if the whole number is a measure of the denominator, divide the denominator by it.

For by *increasing the number* of the parts (*i. e.* multiplying the numerator), or by *increasing the size or value* of the parts (*i. e.* dividing the denominator) the fraction is *increased.*

Thus, $\frac{3}{8} \times 3 = \frac{9}{8}$; since 3 times 3 things, whatever their name, must make 9 such things.

So also $\frac{5}{16} \times 2 = \frac{5}{8}$; for in the fraction $\frac{5}{16}$, the unit is divided into 16 equal parts; and in the fraction $\frac{5}{8}$, into 8 equal parts. Each of these latter parts is manifestly twice as large as each of the former; but the *same number* of the larger parts have been taken; therefore the latter fraction must be twice as great as the former.

2.—*To divide a fraction by a whole number.*

Rule.—Divide the numerator by the whole number, if it can be done without leaving a remainder; if not, multiply the denominator by the whole number.

For by *decreasing the number* of the parts (*i. e.* dividing the numerator), or by *decreasing the size or value* of the parts (*i. e.* multiplying the denominator), the fraction is *decreased.*

Thus, $\frac{9}{11} \div 3 = \frac{3}{11}$; since the third part of 9 articles whatever must be 3 such articles.

So also $\frac{5}{8} \div 2 = \frac{5}{16}$; for the latter fraction expresses the same number of parts of half the size of those in the former fraction; the latter is, therefore, evidently half the value of the former.

Obs.—From this it will be evident that, if *both* the numerator and denominator of a fraction are multiplied or divided by the same number, the *value* of the fraction is unaltered.

For by multiplying both numerator and denominator by the same number, the *number* of the parts is *increased* in exactly the same degree as the *value* of each is *decreased*, and *vice versâ* by division.

3.—*To express a whole number as a fraction with a given denominator.*

Rule.—Multiply the whole number by the given denominator, and the product will be the numerator required.

Thus, 9 expressed with a denominator 5 will be $\frac{45}{5}$; for $9 = \frac{9}{1}$, and $\frac{9 \times 5}{1 \times 5} = \frac{45}{5}$ [Obs. Rule 2].

Or thus, since $1 = \frac{5}{5}$, 9, which is 9 times 1, must equal 9 times $\frac{5}{5}$, that is $\frac{45}{5}$; [Rule 1].

Ex. 42.

(1.) Multiply $\frac{5}{24}$ separately by 2, 4, 5, 6, 8, 11.
(2.) „ $\frac{5}{39}$ „ 3, 5, 7, 9, 12, 13.
(3.) „ $\frac{127}{288}$ „ 6, 8, 12, 16, 36, 72.

(4.) Divide $\frac{316}{275}$ separately by 2, 4, 8, 9, 12, 24.
(5.) „ $\frac{983}{765}$ „ 3, 6, 7, 8, 9, 11.
(6.) „ $\frac{113}{119}$ „ 7, 8, 9, 14, 20, 56.

(7.) Express 8, 13, 25 as fractions with denominators 7, 11, 15.
(8.) „ 13, 15, 29 „ „ „ „ 5, 13, 17.
(9.) „ 18, 23, 58 „ „ „ „ 8, 21, 43.

(10.) How many eighths in 2, 4, 7, 10, 17, 64.
(11.) „ „ thirteenths „ 5, 7, 9, 11, 13, 15.
(12.) „ „ thirtieths „ 6, 10, 15, 30, 45, 100.

(13.) A boy walks $\frac{5}{16}$ of a mile in 6 minutes. How far does he wall in 1 minute, in half an hour, and in 1 hour?

(14.) Seven persons buy $\frac{13}{16}$ of a cwt. of potatoes each. How much is bought in all?

(15.) Nine boys share equally $\frac{27}{40}$ of a sovereign. What is each boy's share?

4.—*To reduce a fraction to its lowest terms.*

A fraction is said to be in its **lowest terms** when its numerator and denominator are *prime to each other.*

RULE.—Divide *both* numerator and denominator *successively* by their common factors, or *at once* by their **G.C.M.** (if the common factors are not readily perceived).

For if the numerator and denominator of a fraction are either multiplied or divided by the *same* number, their *ratio*, or relation to each other, is unaltered, and the value of the fraction remains the same.

Thus, $\frac{5}{10}$, $\frac{4}{8}$, $\frac{3}{6}$, $\frac{2}{4}$, are each equal to $\frac{1}{2}$, the numerator in each fraction being half the denominator.

EXAMPLE.—Reduce $\frac{1296}{1512}$ to its lowest terms.

$$\frac{1296}{1512} = \frac{144}{168} = \frac{48}{56} = \frac{6}{7}.$$

The result is obtained by dividing both terms of the fraction *successively* by their common factors, 9, 3, 8.

The same might have been obtained at once by dividing both terms by their G.C.M., *i. e.* 216. Thus:—

$$\frac{1296 \div 216}{1512 \div 216} = \frac{6}{7}$$

Ex. 43.

Reduce to their lowest terms :—

(1.) $\frac{48}{54}$, $\frac{63}{72}$, $\frac{99}{144}$.
(2.) $\frac{30}{105}$, $\frac{70}{105}$, $\frac{196}{200}$.
(3.) $\frac{112}{180}$, $\frac{84}{196}$, $\frac{132}{240}$.
(4.) $\frac{324}{396}$, $\frac{415}{575}$, $\frac{686}{864}$.
(5.) $\frac{315}{378}$, $\frac{528}{594}$, $\frac{756}{972}$.
(6.) $\frac{784}{804}$, $\frac{894}{990}$, $\frac{321}{749}$.
(7.) $\frac{249}{415}$, $\frac{665}{950}$, $\frac{357}{459}$.
(8.) $\frac{858}{936}$, $\frac{475}{1000}$, $\frac{35}{945}$.
(9.) $\frac{288}{320}$, $\frac{535}{856}$, $\frac{407}{444}$.
(10.) $\frac{2805}{3135}$, $\frac{2500}{4181}$.
(11.) $\frac{4117}{9487}$, $\frac{2484}{4554}$.
(12.) $\frac{3157}{3276}$, $\frac{1850}{3034}$.
(13.) $\frac{37444}{429000}$, $\frac{23779}{23586}$.
(14.) $\frac{511}{10001}$, $\frac{1554}{10098}$.
(15.) $\frac{3575}{4719}$, $\frac{9604}{7889}$
(16.) $\frac{777777}{854700}$, $\frac{632888}{731296}$.
(17.) $\frac{527751}{645029}$, $\frac{32277}{81620}$.
(18.) $\frac{857142}{999999}$, $\frac{10758}{40815}$.

5.—*To reduce a mixed number to an improper fraction.*

Rule.—Multiply the whole number by the denominator of the fractional part, add the numerator to the product, and place the denominator under the sum so obtained.

Example.—Reduce $7\frac{3}{4}$ to an improper fraction.

$$7\frac{3}{4} = \frac{7 \times 4 + 3}{4} = \frac{31}{4}.$$

$$\text{for } 7\frac{3}{4} = 7 + \frac{3}{4} = \frac{28}{4} + \frac{3}{4} = \frac{31}{4}.$$

Ex. 44.

Reduce the following mixed numbers to improper fractions :—

(1.) $4\frac{1}{6}$, $7\frac{2}{11}$, $9\frac{3}{8}$, $10\frac{9}{10}$.
(2.) $7\frac{5}{16}$, $24\frac{1}{18}$, $29\frac{8}{15}$, $31\frac{13}{17}$.
(3.) $3\frac{16}{113}$, $13\frac{6}{7}$, $8\frac{4}{15}$, $17\frac{15}{16}$.
(4.) $13\frac{6}{17}$, $11\frac{2}{15}$, $71\frac{11}{12}$, $28\frac{11}{26}$.
(5.) $119\frac{37}{50}$, $172\frac{12}{17}$, $514\frac{5}{16}$.
(6.) $111\frac{99}{110}$, $901\frac{15}{16}$, $982\frac{12}{35}$.
(7.) $13\frac{329}{400}$, $23\frac{851}{1080}$, $34\frac{1139}{1440}$.
(8.) $216\frac{149}{401}$, $611\frac{131}{200}$, $759\frac{128}{333}$.
(9.) How many quarter-pounds in $14\frac{3}{4}$ lbs. ?
(10.) How many twelfths of a foot in $36\frac{5}{12}$ feet ?
(11.) How many eighths of a mile in 13 lea. $2\frac{3}{8}$ miles ?
(12.) How many thirds of a yard in 4 miles 7 fur. $37\frac{2}{3}$ yards ?

6.—*To reduce an improper fraction to a whole or mixed number.*

Rule.—Divide the numerator by the denominator, and set down the quotient as a whole number : the remainder, if any, will be the numerator of the fractional part of the mixed number, retaining the same denominator.

Example.—Reduce $\frac{9}{4}$ and $\frac{275}{12}$ to mixed numbers.

$$\frac{9}{4} = 9 \div 4 = 2\tfrac{1}{4}; \quad \frac{275}{12} = 275 \div 12 = 24\tfrac{11}{12}.$$

Ex. 45.

(1.) $\frac{26}{7}$, $\frac{92}{9}$, $\frac{94}{11}$, $\frac{65}{14}$.
(2.) $\frac{101}{17}$, $\frac{100}{31}$, $\frac{119}{7}$, $\frac{117}{16}$.
(3.) $\frac{236}{17}$, $\frac{427}{13}$, $\frac{263}{12}$, $\frac{228}{115}$.
(4.) $\frac{339}{60}$, $\frac{908}{72}$, $\frac{2000}{73}$, $\frac{289}{17}$.
(5.) $\frac{7726}{331}$, $\frac{8264}{89}$, $\frac{98763}{471}$.
(6.) $\frac{3218}{360}$, $\frac{12421}{111}$, $\frac{3149}{125}$.
(7.) $\frac{2008}{155}$, $\frac{2497}{119}$, $\frac{10301}{101}$.
(8.) $\frac{8869}{587}$, $\frac{110667}{1125}$, $\frac{111203992}{1101}$.

(9.) How many yards in 327 quarter-yards?

(10.) A person receives $\frac{571}{16}$ of a pound. How many pounds does he receive?

(11.) How many years in $\frac{1461}{4}$ days?

(12.) How many whole cakes in $\frac{472}{12}$ of a cake?

7.—*To express fractions having different denominators by equivalent fractions having a common denominator.*

Before quantities of one denomination can be added to, subtracted from, or compared with quantities of another denomination, they must be expressed in some common denomination.

Thus, it is impossible to take 6 lbs. from 6 yards, because the quantities have nothing in common. Again, 6 sheep and 4 cows *are neither* 10 sheep *nor* 10 cows, but *they are* 10 *animals*, animals being a name common to both. Once more, before we can compare the value of a number of different coins, we must express the value of each in some common standard, as shillings, pence, or pounds. The same thing is true of fractions; therefore, before they can be compared together, or combined into one result, they must be of the same name, that is, they must have the same denominator. This is obtained by the following rule, which gives the **Least Common Denominator.**

Rule.—Find the **L.C.M.** of all the denominators, and take this for the common denominator; then, for the new numerators, multiply the numerator of each fraction by the quotient obtained by dividing the common denominator by the denominator of that same fraction.

XAMPLE.—Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, to equivalent fractions, having the ast Common Denominator.

The L.C.M. of 3, 4, 6, 8 is 24, which is the Least Common Denomi- tor.

$$\tfrac{2}{3} = \frac{2 \times 8}{24} = \tfrac{16}{24}.$$

$$\tfrac{3}{4} = \frac{3 \times 6}{24} = \tfrac{18}{24}.$$

$$\tfrac{5}{6} = \frac{5 \times 4}{24} = \tfrac{20}{24}.$$

$$\tfrac{7}{8} = \frac{7 \times 3}{24} = \tfrac{21}{24}.$$

Or worked thus:—

$24 \div 3 = 8$; $8 \times 2 = 16$.
$24 \div 4 = 6$; $6 \times 3 = 18$.
$24 \div 6 = 4$; $5 \times 4 = 20$.
$24 \div 8 = 3$; $7 \times 3 = 21$

Hence the equivalent fractions are $\frac{16}{24}$, $\frac{18}{24}$, $\frac{20}{24}$, $\frac{21}{24}$.

In this process, *both* the numerator and denominator of each fraction have been multiplied by the *same* number (viz. that which would make its denominator 24), and, therefore, the value of the fraction remains unaltered.

Ex. 46.

Express the following fractions by equivalent ones, with the Least Common Denominator:—

(1.) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{9}$.
(2.) $\frac{3}{4}$, $\frac{2}{5}$, $\frac{1}{8}$, $\frac{7}{16}$.
(3.) $\frac{2}{3}$, $\frac{3}{7}$, $\frac{7}{10}$, $\frac{13}{21}$.
(4.) $\frac{3}{8}$, $\frac{5}{6}$, $\frac{7}{12}$, $\frac{5}{48}$.
(5.) $\frac{5}{9}$, $\frac{7}{18}$, $\frac{11}{36}$, $\frac{13}{54}$.
(6.) $\frac{9}{10}$, $\frac{10}{11}$, $\frac{13}{121}$, $\frac{17}{110}$.
(7.) $\frac{3}{7}$, $\frac{19}{21}$, $\frac{47}{56}$, $\frac{61}{63}$.
(8.) $\frac{3}{16}$, $\frac{7}{48}$, $\frac{5}{64}$, $\frac{37}{96}$.
(9.) $\frac{17}{81}$, $\frac{13}{27}$, $\frac{70}{99}$, $\frac{97}{108}$.
(10.) $\frac{13}{243}$, $\frac{13}{135}$, $\frac{13}{729}$, $\frac{13}{405}$.
(11.) $\frac{17}{45}$, $\frac{13}{48}$, $\frac{117}{225}$, $\frac{59}{96}$.
(12.) $\frac{13}{25}$, $\frac{124}{125}$, $\frac{624}{625}$, $\frac{898}{975}$.
(13.) $\frac{3}{5}$, $\frac{7}{25}$, $\frac{27}{45}$, $\frac{11}{15}$, $\frac{13}{75}$.
(14.) $\frac{11}{12}$, $\frac{11}{24}$, $\frac{11}{60}$, $\frac{11}{96}$, $\frac{11}{120}$
(15.) $\frac{7}{30}$, $\frac{5}{8}$, $\frac{8}{9}$, $\frac{43}{48}$, $\frac{15}{16}$.
(16.) $\frac{19}{42}$, $\frac{4}{7}$, $\frac{27}{35}$, $\frac{23}{84}$, $\frac{17}{49}$.
(17.) $\frac{17}{18}$, $\frac{5}{12}$, $\frac{35}{102}$, $\frac{8}{63}$, $\frac{7}{18}$.
(18.) $\frac{10}{17}$, $\frac{21}{68}$, $\frac{95}{119}$, $\frac{131}{136}$, $\frac{11}{34}$
(19.) $\frac{2}{3}$, $\frac{3}{7}$, $\frac{3}{11}$, $\frac{5}{13}$, $\frac{7}{35}$.
(20.) $\frac{5}{6}$, $\frac{13}{18}$, $\frac{7}{9}$, $\frac{11}{12}$, $\frac{23}{36}$.

XV.—ADDITION OF VULGAR FRACTIONS.

Rule.—Express the fractions by equivalent ones, with the Least Common Denominator. Add the numerators together, and retain the common denominator.

Example.—Add together (i) $\frac{2}{3}$, $\frac{1}{6}$, $\frac{5}{9}$, $\frac{11}{12}$; (ii.) $\frac{5}{7}$, $\frac{2}{15}$, $\frac{7}{9}$, $\frac{11}{21}$.

(i.) $\frac{2}{3} + \frac{1}{6} + \frac{5}{9} + \frac{11}{12} = \frac{24}{36} + \frac{6}{36} + \frac{20}{36} + \frac{33}{36} = \frac{83}{36} = 2\frac{11}{36}$.

(ii.) $\frac{5}{7} + \frac{2}{15} + \frac{7}{9} + \frac{11}{21} = \dfrac{225 + 42 + 245 + 165}{315} = \frac{677}{315} = 2\frac{47}{315}$.

When any of the quantities to be added are whole or mixed numbers, the whole numbers should be added first, and then the fractions.

Example.—Add together (i.) $2\frac{1}{3}$, $4\frac{1}{5}$, $5\frac{1}{4}$; (ii.) $3\frac{6}{7}$, $11\frac{11}{14}$, $7\frac{15}{28}$.

(i.) $2\frac{1}{3} + 4\frac{1}{5} + 5\frac{1}{4} = 11 + \dfrac{20 + 12 + 15}{60} = 11\frac{47}{60}$.

(ii.) $3\frac{6}{7} + 11\frac{11}{14} + 7\frac{15}{28} = 21 + \dfrac{24 + 22 + 15}{28} = 21 + 2\frac{5}{28} = 23\frac{5}{28}$.

It is best to reduce improper fractions to mixed numbers before adding them. Fractions should always be expressed in their lowest terms.

Ex. 47.

(1.) $\frac{3}{8} + \frac{5}{16}$
$\frac{4}{15} + \frac{7}{30}$
$\frac{4}{15} + \frac{11}{60}$
$\frac{3}{17} + \frac{4}{51}$

(2.) $\frac{3}{7} + \frac{5}{28}$
$\frac{7}{25} + \frac{11}{75}$
$\frac{14}{15} + \frac{13}{75}$
$\frac{3}{14} + \frac{11}{28}$

(3.) $\frac{7}{10} + \frac{3}{20}$
$\frac{5}{16} + \frac{1}{24}$
$\frac{11}{28} + \frac{13}{112}$
$\frac{13}{17} + \frac{53}{68}$

(4.) $\frac{5}{12} + \frac{7}{24}$
$\frac{5}{18} + \frac{17}{36}$
$\frac{18}{63} + \frac{11}{126}$
$\frac{12}{19} + \frac{52}{57}$

(5.) $\frac{15}{16} + \frac{14}{48}$
$\frac{12}{13} + \frac{27}{39}$
$\frac{21}{23} + \frac{19}{69}$
$\frac{11}{15} + \frac{17}{45}$

(6.) $\frac{13}{17} + \frac{40}{51}$
$\frac{14}{15} + \frac{17}{30}$
$\frac{14}{25} + \frac{117}{125}$
$\frac{15}{19} + \frac{49}{76}$

(7.) $\frac{13}{21} + \frac{25}{28} + \frac{71}{84}$
$\frac{11}{35} + \frac{71}{105} + \frac{11}{21}$
$\frac{7}{27} + \frac{40}{54} + \frac{107}{108}$
$\frac{7}{12} + \frac{11}{48} + \frac{13}{96}$

(8.) $\frac{11}{14} + \frac{41}{56} + \frac{93}{112}$
$\frac{11}{15} + \frac{17}{60} + \frac{113}{120}$
$\frac{12}{35} + \frac{7}{15} + \frac{13}{105}$
$\frac{13}{15} + \frac{11}{25} + \frac{117}{125}$

(9.) $\frac{71}{84} + \frac{11}{16} + \frac{19}{112}$
$\frac{15}{17} + \frac{11}{51} + \frac{13}{102}$
$\frac{11}{18} + \frac{17}{54} + \frac{11}{108}$
$\frac{13}{36} + \frac{17}{72} + \frac{19}{144}$

(10.) $\frac{7}{15} + \frac{13}{35} + \frac{17}{105}$
$\frac{5}{12} + \frac{27}{52} + \frac{9}{13}$
$\frac{10}{17} + \frac{21}{68} + \frac{57}{136}$
$\frac{17}{15} + \frac{23}{75} + \frac{47}{65}$

(11.) $\frac{7}{15} + \frac{11}{35} + \frac{17}{21} + \frac{3}{7}$
$\frac{3}{7} + \frac{5}{21} + \frac{11}{42} + \frac{17}{24}$
$\frac{7}{12} + \frac{5}{24} + \frac{17}{36} + \frac{5}{18}$
$\frac{5}{9} + \frac{17}{63} + \frac{17}{21} + \frac{11}{18}$

(12.) $\frac{7}{25} + \frac{4}{75} + \frac{17}{45} + \frac{8}{9}$
$\frac{11}{72} + \frac{13}{24} + \frac{14}{15} + \frac{7}{60}$
$\frac{3}{14} + \frac{5}{7} + \frac{11}{28} + \frac{17}{56}$
$\frac{17}{27} + \frac{17}{63} + \frac{10}{21} + \frac{13}{18}$

(13.) $\frac{11}{12} + \frac{5}{63} + \frac{73}{81} + \frac{7}{24} + \frac{11}{27}$
$\frac{14}{15} + \frac{13}{70} + \frac{19}{30} + \frac{17}{35} + \frac{4}{21}$
$\frac{13}{14} + \frac{65}{72} + \frac{15}{28} + \frac{7}{12} + \frac{11}{84}$
$\frac{41}{80} + \frac{13}{70} + \frac{23}{560} + \frac{53}{140} + \frac{67}{420}$

(14.) $\frac{3}{7} + \frac{16}{49} + \frac{17}{21} + \frac{1}{3} + \frac{11}{63}$
$\frac{17}{21} + \frac{11}{12} + \frac{17}{84} + \frac{5}{8} + \frac{15}{48}$
$\frac{5}{9} + \frac{13}{24} + \frac{11}{72} + \frac{13}{108} + \frac{17}{36}$
$\frac{1}{9} + \frac{3}{5} + \frac{7}{15} + \frac{1}{3} + \frac{5}{27}$

(15.) $\frac{3}{12} + \frac{7}{16} + \frac{3}{24} + \frac{5}{6} + \frac{3}{4}$
$\frac{17}{20} + \frac{7}{10} + \frac{2}{5} + \frac{13}{25} + \frac{19}{40}$
$\frac{11}{18} + \frac{16}{39} + \frac{2}{3} + \frac{5}{78} + \frac{31}{52}$
$\frac{3}{7} + \frac{11}{14} + \frac{17}{32} + \frac{19}{28} + \frac{13}{112}$

(16.) $\frac{2}{5} + \frac{9}{70} + \frac{7}{20} + \frac{3}{140} + \frac{3}{2800}$
$\frac{1}{8} + \frac{1}{63} + \frac{1}{560} + \frac{1}{5040} + \frac{1}{840}$
$\frac{4}{17} + \frac{7}{50} + \frac{21}{850} + \frac{11}{90} + \frac{13}{180}$
$\frac{15}{16} + \frac{14}{15} + \frac{13}{14} + \frac{11}{12} + \frac{9}{10}$

(17) $2\frac{1}{3} + 4\frac{1}{5} + 5\frac{1}{4}$; $2\frac{1}{5} + 5\frac{1}{7} + 7\frac{4}{35}$; $3\frac{6}{7} + 11\frac{11}{14} + 7\frac{15}{28}$; $5\frac{1}{9} + 12\frac{7}{45} + 7\frac{5}{63}$.

(18.) $5\frac{4}{13} + 7\frac{5}{26} + 8\frac{5}{75}$; $18\frac{16}{45} + 9\frac{7}{8} - 25\frac{5}{12}$; $17\frac{1}{42} + 3\frac{5}{21} + 18\frac{31}{63}$ $13\frac{3}{40} + 20\frac{19}{72} + 3\frac{7}{18}$.

(19.) $3\frac{11}{12} + 5\frac{14}{15} + 7\frac{20}{27} + 3\frac{3}{40}$; $8\frac{1}{7} + 5\frac{1}{21} + 7\frac{4}{15} + 12\frac{1}{14}$; $15\frac{3}{14} + 7\frac{5}{12} \div 3\frac{7}{15} + 13\frac{7}{10}$.

(20) $14\frac{3}{11} + 7\frac{5}{36} + 12\frac{5}{44} + 15\frac{5}{108} + 8\frac{3}{56} + 7\frac{17}{30} + 11\frac{10}{77}$; $18\frac{5}{8} + 7\frac{3}{10} + 11\frac{7}{12} + 3\frac{17}{21} + 12\frac{11}{35} + 5\frac{17}{63} + 12\frac{2}{45}$.

XVI.—SUBTRACTION OF FRACTIONS.

Rule.—Express the fractions by equivalent ones, with the Least Common Denominator; find the difference of the numerators, and retain the common denominator.

Example.—Subtract $\frac{6}{7}$ from $\frac{8}{9}$.

$$\frac{8}{9} - \frac{6}{7} = \frac{56 - 54}{63} = \frac{2}{63}.$$

When the quantities whose difference is required are whole or mixed numbers, the difference of the whole numbers should be found first, and then the difference of the fractions; and the two results combined, or the mixed numbers may be reduced to improper fractions, and subtracted according to the preceding rule.

EXAMPLE.—Find the difference between $4\frac{11}{12}$ and $2\frac{5}{8}$; also between $3\frac{3}{4}$ and $7\frac{2}{3}$.

(i.) $4\frac{11}{12} - 2\frac{5}{8} = 2 + \frac{22 - 15}{24} = 2\frac{7}{24}$.

(ii.) $7\frac{2}{3} - 3\frac{3}{4} = 4 + \frac{8 - 9}{12} = 4 - \frac{1}{12} = 3\frac{11}{12}$;

Or thus: $7\frac{2}{3} - 3\frac{3}{4} = \frac{23}{3} - \frac{15}{4} = \frac{92 - 45}{12} = \frac{47}{12} = 3\frac{11}{12}$.

Ex. 48.

(1.) $\frac{3}{7} - \frac{1}{8}$	(2.) $\frac{10}{11} - \frac{7}{13}$	(3.) $\frac{11}{12} - \frac{2}{5}$
$\frac{2}{3} - \frac{3}{7}$	$\frac{11}{13} - \frac{11}{14}$	$\frac{7}{9} - \frac{11}{36}$
$\frac{5}{9} - \frac{2}{7}$	$\frac{7}{15} - \frac{13}{35}$	$\frac{29}{60} - \frac{7}{15}$
$\frac{5}{6} - \frac{4}{7}$	$\frac{17}{64} - \frac{5}{56}$	$\frac{41}{45} - \frac{28}{75}$
(4.) $\frac{5}{17} - \frac{3}{34}$	(5.) $\frac{19}{21} - \frac{17}{49}$	(6.) $\frac{16}{21} - \frac{17}{35}$
$\frac{19}{20} - \frac{13}{17}$	$\frac{14}{15} - \frac{41}{50}$	$\frac{11}{16} - \frac{33}{128}$
$\frac{5}{18} - \frac{13}{54}$	$\frac{17}{19} - \frac{23}{57}$	$\frac{16}{53} - \frac{15}{52}$
$\frac{21}{22} - \frac{20}{21}$	$\frac{29}{34} - \frac{39}{119}$	$\frac{99}{112} - \frac{59}{84}$

(7.) $1\frac{17}{24} - \frac{7}{12}$; $7\frac{4}{5} - 5\frac{13}{35}$; $16\frac{3}{4} - 11\frac{3}{5}$; $13\frac{4}{5} - 12\frac{3}{10}$.

(8.) $3\frac{1}{3} - 2\frac{1}{4}$; $5\frac{3}{8} - 2\frac{1}{3}$; $6\frac{4}{7} - 3\frac{9}{14}$; $23\frac{7}{12} - 19\frac{13}{48}$.

(9.) $3\frac{4}{25} - 1\frac{3}{4}$; $5\frac{3}{14} - 2\frac{5}{21}$; $1\frac{8}{35} - \frac{5}{6}$; $13\frac{9}{10} - 9\frac{17}{55}$.

(10.) $3\frac{3}{7} - 2\frac{5}{9}$; $5\frac{13}{24} - 4\frac{17}{18}$; $14\frac{19}{20} - 11\frac{15}{16}$; $27\frac{17}{32} - 16\frac{19}{24}$.

(11.) $2\frac{13}{15} + 3\frac{9}{11} - 4\frac{5}{12}$; $6\frac{7}{11} + 1\frac{9}{35} - 5\frac{8}{9}$; $5\frac{7}{8} - 6\frac{3}{4} + 1\frac{5}{6}$.

(12.) $8\frac{5}{12} - 3\frac{9}{10} - 2\frac{7}{15}$; $\frac{14}{21} + 5\frac{4}{7} - 6\frac{2}{15}$; $10\frac{14}{15} - 7\frac{8}{35} - 2\frac{5}{12}$.

(13.) $4\frac{13}{55} - 2\frac{7}{11} - 1\frac{4}{5}$; $3\frac{5}{6} - 1\frac{4}{7} + 5\frac{8}{9}$; $9\frac{17}{18} - 7\frac{2}{3} - 1\frac{2}{5}$.

(14.) $11\frac{13}{16} - 6\frac{7}{12} - 2\frac{4}{3} - 1\frac{1}{5}$; $4\frac{17}{5} + \frac{9}{11} - \frac{7}{18} - 3\frac{3}{10}$; $5\frac{13}{3} - 2\frac{7}{15} + 3\frac{5}{18} + \frac{1}{2}$.

(15.) $7\frac{1}{4} - 5\frac{1}{2} - \frac{7}{24} - \frac{15}{16}$; $13\frac{13}{22} - 12\frac{2}{11} - \frac{3}{8} - \frac{1}{12}$; $15\frac{7}{9} + 1\frac{1}{3} - 14\frac{11}{12} + \frac{3}{5}$

XVII.—MULTIPLICATION OF FRACTIONS.

MULTIPLICATION by *whole numbers* is the method of finding the amount of a number or quantity repeated a certain number of times.

MULTIPLICATION by *fractions* is so far like ordinary multiplication, that it signifies *repetitions*; it is not, how-he *whole* quantity, but only *a part* of it, that is to be ed. The *part* to be taken is shown by the *denomina-* the multiplier, and the number of *repetitions*, by *umerator.*

, to multiply $\frac{3}{5}$ by $\frac{2}{7}$ means that the *seventh part* of $\frac{3}{5}$ is to be *twice;* but the seventh part of $\frac{3}{5}$ is $\frac{3}{35}$ [Page 82, § 2], and $\frac{3}{5}$ is manifestly $\frac{6}{35}$; therefore $\frac{3}{5} \times \frac{2}{7} = \frac{6}{35}$.

we see that the product of the two fractions is equivalent to the t of the numerators divided by the product of the denominators.

n, $\frac{3}{5} \times \frac{2}{7}$ may be written as $\frac{3}{5}$ of $\frac{2}{7}$, or $\frac{2}{7}$ of $\frac{3}{5}$. Expressions of er form have been termed **Compound Fractions**, or *fractions of s*, and are simplified by the same rule of Multiplication.

LE.—Multiply all the numerators for a new numerator, 1 the denominators for a new denominator.

d numbers must first be reduced to improper fractions.

process of Multiplication may frequently be simplified by can-, that is, by striking out all factors common to numerator and inator.

ncelling, *any* numerator and denominator may be divided by a *factor*, the number of times it is contained being set down he numerator and below the denominator.

$$\text{Thus :—}\ \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\underset{2}{\cancel{4}}} = \frac{1}{2};$$

n a numerator or denominator is entirely cancelled, *unity*, or 1, akes its place, but is not usually written : thus, in the previous e, 3 is contained in 3 *once ;* 2 is contained in 2 *once*, and in 4 the 1 is not written, but the 2 is.

MPLE I.—Multiply together $\frac{3}{5}$, $\frac{4}{7}$, $\frac{6}{11}$.

$$\frac{3}{5} \times \frac{4}{7} \times \frac{6}{11} = \frac{3 \times 4 \times 6}{5 \times 7 \times 11} = \frac{72}{385}.$$

MPLE II.—Simplify $\frac{3}{4}$ of $\frac{7}{9}$ of $\frac{4}{5}$ of $\frac{3}{8}$ of $\frac{9}{10}$ of $\frac{6}{7}$.

$$\frac{3}{\cancel{4}} \times \frac{\cancel{7}}{\cancel{9}} \times \frac{\cancel{4}}{5} \times \frac{3}{\underset{4}{\cancel{8}}} \times \frac{\cancel{9}}{10} \times \frac{\overset{3}{\cancel{6}}}{\cancel{7}} = \frac{27}{200}.$$

Here 7, 4, and 9, appearing in both numerator and denominator, are cancelled: then, since 2 is contained in 6, three times, and in 8, four times, the 6 is struck out, and 3 placed above it, and the 8 struck out, and 4 written below it. This depends on the principle, "That, if numerator and denominator be divided by the same number, the *value* of the fraction is unaltered."

$$\text{Then, } \frac{3 \times 3 \times 3}{5 \times 4 \times 10} = \frac{27}{200}.$$

Example III.—Multiply together $3\frac{1}{3} \times \frac{5}{7}$ of $4\frac{2}{3} \times 3\frac{3}{8}$.

$$3\tfrac{1}{3} \times \tfrac{5}{7} \times 4\tfrac{2}{3} \times 3\tfrac{3}{8} = \frac{\overset{2}{\cancel{16}}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{7}} \times \frac{\overset{2}{\cancel{14}}}{\cancel{3}} \times \frac{\overset{9}{\cancel{27}}}{\cancel{8}} = \frac{36}{1} = 36.$$

Ex. 49.

(1.) $\frac{4}{15} \times \frac{7}{24} \times \frac{25}{49}$.

(2.) $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{15}{22}$.

(3.) $\frac{1}{3}$ of $\frac{5}{7} \times \frac{6}{11} \times \frac{14}{25}$.

(4.) $\frac{3}{4} \times \frac{7}{9} \times \frac{3}{8}$ of $\frac{16}{21}$

(5.) $3\frac{2}{7} \times 2\frac{21}{22} \times 2\frac{1}{5}$.

(6.) $3\frac{3}{4} \times 4\frac{4}{15} \times 3\frac{1}{8}$

(7.) $5\frac{1}{4} \times 3\frac{3}{4} \times \frac{4}{13}$ of $\frac{3}{7}$.

(8.) $4\frac{1}{5} \times 4\frac{2}{7} \times 2\frac{7}{9}$.

(9.) $6\frac{1}{4} \times 3\frac{1}{7} \times 3\frac{4}{15} \times 2\frac{3}{11}$.

(10.) $3\frac{5}{9}$ of $1\frac{5}{16}$ of $1\frac{2}{7} \times 4\frac{1}{6}$.

(11.) $161\frac{7}{8} \times 4\frac{4}{5} \times \frac{3}{7}$ of $\frac{8}{111}$.

(12.) $15\frac{5}{9} \times 5\frac{1}{7} \times 1\frac{1}{20}$.

(13.) $21\frac{3}{5} \times 7\frac{7}{8} \times 2\frac{2}{7} \times \frac{25}{351}$.

(14.) $2\frac{1}{12} \times 2\frac{1}{3} \times 5\frac{9}{10} \times 1\frac{13}{59}$.

(15.) $66\frac{7}{9} \times \frac{13}{14} \times 5\frac{3}{5}$.

(16.) $\frac{19}{20}$ of $11\frac{1}{9} \times 3\frac{21}{38}$.

(17.) $3\frac{5}{11} \times 4\frac{1}{4}$ of $\frac{4}{35} \times \frac{11}{17}$ of $8\frac{3}{4}$.

(18.) $\frac{35}{36}$ of $7\frac{1}{12} \times \frac{90}{91}$ of $\frac{3}{17}$ of $7\frac{1}{5}$.

XVIII.—DIVISION OF FRACTIONS.

Division of Fractions, like division in whole numbers, is the mode of finding how often one number is contained in another, or what multiple the dividend is of the divisor.

The term "multiple" is here used in an extended sense, including any "part or parts."

To divide $\frac{5}{11}$ by $\frac{2}{7}$ means to find, either

(i.) How often $\frac{2}{7}$ is contained in $\frac{5}{11}$; or

(ii.) What number multiplied by $\frac{2}{7}$ will produce $\frac{5}{11}$; or

(iii.) What part $\frac{5}{11}$ is of $\frac{2}{7}$.

Now $\frac{5}{11} \div \frac{2}{7}$ is manifestly the same as $\frac{35}{77} \div \frac{22}{77}$ (since both have been reduced to equivalent fractions with a common denominator), and 35 divided by 22 equals $1\frac{13}{22}$, whatever the 35 or 22 may represent;

$\div \frac{2}{7} = \frac{35}{22} = 1\frac{13}{22}$, which is the same number as would result $\times \frac{7}{2}$, or from multiplying the dividend by the divisor *inverted.*

same result may be reasoned out thus:—

$\div 2 = \frac{1}{2}$; $\frac{1}{11} \div 2 = \frac{1}{22}$; $\frac{5}{11} \div 2 = \frac{5}{22}$; $\frac{5}{11} \div \frac{2}{7} = \frac{35}{22}$.

divided by two equals $\frac{1}{2}$.

e-eleventh divided by two equals $\frac{1}{11}$ of $\frac{1}{2} = \frac{1}{22}$.

ve-elevenths divided by two equal $5 \times \frac{1}{22} = \frac{5}{22}$.

ve-elevenths divided by two-sevenths equal $7 \times \frac{5}{22} = \frac{35}{22}$; for the sor ($\frac{2}{7}$), being only $\frac{1}{7}$ of the former divisor (2), the quotient must seven times as great.

Rule.—Invert the divisor, and then proceed as in multi-ication.

Mixed numbers must be reduced to improper fractions.

If the divisor be itself a *compound fraction*, each of the fractions composing it must be inverted.

Example.—Divide (i.) $\frac{9}{11}$ by $\frac{5}{8}$; (ii.) $3\frac{3}{4}$ by $2\frac{2}{3}$; (iii.) $4\frac{1}{2}$ of $\frac{5}{7}$ by $3\frac{1}{4}$ of $\frac{9}{13}$.

(i.) $\frac{9}{11} \div \frac{5}{8} = \frac{9}{11} \times \frac{8}{5} = \frac{72}{55} = 1\frac{17}{55}$.

(ii.) $3\frac{3}{4} \div 2\frac{2}{3} = \frac{15}{4} \div \frac{8}{3} = \frac{15}{4} \times \frac{3}{8} = \frac{45}{32} = 1\frac{13}{32}$.

$$\text{(iii.) } (4\tfrac{1}{2} \times \tfrac{5}{7}) \div (3\tfrac{1}{4} \times \tfrac{9}{13}) = \frac{\cancel{9}}{\cancel{2}} \times \frac{5}{7} \times \frac{\overset{2}{\cancel{4}}}{\cancel{13}} \times \frac{\cancel{13}}{\cancel{9}} = \frac{10}{7} = 1\tfrac{3}{7}.$$

Fractions in which either the numerator, or the denominator, or both, are fractions, are termed **Complex Fractions.**

They may be simplified by the rule of Division.

Example.—Simplify (i.) $\dfrac{\frac{5}{8}}{\frac{15}{16}}$; (ii.) $\dfrac{3\frac{1}{3}}{2\frac{5}{8}}$; (iii.) $\dfrac{2\frac{1}{3} + 1\frac{4}{5}}{3\frac{4}{5} - 2\frac{1}{3}}$.

$$\text{(i.) } \frac{\frac{5}{8}}{\frac{15}{16}} = \frac{5}{8} \div \frac{15}{16} = \frac{\cancel{5}}{\cancel{8}} \times \frac{\overset{2}{\cancel{16}}}{\underset{3}{\cancel{15}}} = \frac{2}{3}.$$

$$\text{(ii.) } \frac{3\frac{1}{3}}{2\frac{5}{8}} = 3\tfrac{1}{3} \div 2\tfrac{5}{8} = \tfrac{10}{3} \div \tfrac{21}{8} = \tfrac{10}{3} \times \tfrac{8}{21} = \tfrac{80}{63} = 1\tfrac{17}{63}.$$

Complex fractions, such as $\frac{2\frac{1}{2}+1\frac{2}{3}}{3\frac{2}{3}-2\frac{1}{2}}$, may be simplified by the previous rules, or, more readily, by multiplying all the terms by the least common multiple of the denominators of the smaller fractions; thus:—

Multiplying each term of the above by 6,

$$\text{(iii)}\quad \frac{2\frac{1}{2}+1\frac{2}{3}}{3\frac{2}{3}-2\frac{1}{2}} = \frac{15+10}{22-15} = \tfrac{25}{7} = 3\tfrac{4}{7}.$$

Ex. 50.

(1.) $\frac{2}{3} \div \frac{1}{4}$; $\frac{12}{13} \div \frac{2}{5}$. (2.) $\frac{7}{12} \div \frac{3}{4}$; $\frac{12}{35} \div \frac{6}{7}$.

(3.) $14\frac{1}{2} \div 4\frac{1}{7}$; $14\frac{7}{8} \div 17$. (4.) $4\frac{37}{63} \div 41\frac{2}{7}$; $52\frac{8}{9} \div 3\frac{1}{2}$.

(5.) $9\frac{11}{13} \div 3\frac{5}{7}$; $11\frac{2}{3} \div 6\frac{5}{8}$. (6.) $\frac{189}{200} \div \frac{3}{5}$ of $2\frac{1}{10}$; $39 \div 9\frac{5}{11}$.

(7.) $7\frac{2}{5}$ of $8\frac{1}{10} \times 11\frac{1}{9} \div 15\frac{6}{7}$. (8.) $21\frac{3}{4} \div 5\frac{3}{19}$ of $5\frac{40}{49}$; $72\frac{1}{2} \div 7\frac{1}{4}$.

(9.) $\frac{2}{3}$ of $\frac{7}{11}$ of $19 \div \frac{3}{7}$ of 5. (10.) $2\frac{2}{5}$ of $\frac{5}{7} \div \frac{6}{7}$ of $4\frac{1}{2}$; $32 \div 1\frac{13}{23}$.

(11.) $34\frac{49}{264} \div (11\frac{1}{9} \times 3\frac{21}{55})$ (12.) $\frac{3}{11}$ of $(14\frac{3}{10} \times 6\frac{10}{13}) \div (\frac{1}{17} \times 229\frac{1}{2})$.

Ex. 51.

Simplify the following fractions:—

(1.) $(22\frac{2}{11} \times 4\frac{2}{5}) - (19\frac{2}{7} \div 4\frac{3}{8})$

$$= \left(\frac{244}{\cancel{11}} \times \frac{\overset{2}{\cancel{22}}}{5}\right) - \left(\frac{\overset{27}{\cancel{135}}}{7} \times \frac{8}{\underset{7}{\cancel{35}}}\right).$$

$$= \frac{488}{5} - \frac{216}{49} = 97\tfrac{3}{5} - 4\tfrac{20}{49}.$$

$$= 93\tfrac{147-100}{245} = 93\tfrac{47}{245}.$$

(2.) $$\left(\frac{1}{4\frac{1}{3}} \times \frac{2\frac{3}{13}}{9}\right) \div \left\{\left(\frac{1}{1\frac{7}{461}} - \frac{1\frac{32}{39}}{2}\right) \times \frac{7\frac{1}{5}}{4}\right\}$$

$$= \left(\frac{3}{13} + \frac{29}{117}\right) \div \left\{\left(\frac{461}{468} - \frac{71}{78}\right) \times \frac{36}{20}\right\}$$

$$= \frac{27+29}{117} \div \left\{\frac{461-426}{468} \times \frac{9}{5}\right\}$$

$$= \frac{\overset{8}{\cancel{56}}}{\cancel{117}} \times \frac{\overset{4}{\cancel{468}}}{\underset{5}{\cancel{35}}} \times \frac{\cancel{5}}{9} = \frac{32}{9} = 3\frac{5}{9}.$$

(1.) $\frac{3\frac{5}{6}}{5\frac{3}{14}} \div \frac{23}{1\frac{2}{3} \text{ of } \frac{3}{5} \text{ of } 21}$; $\frac{5}{16}$ of $\frac{2\frac{4}{7}}{5\frac{3}{8}}$ of $\frac{9\frac{5}{9}}{5\frac{1}{7}}$.

(2.) $(2\frac{3}{4}$ of $\frac{5}{6}$ of $1\frac{2}{3}$ of $2\frac{1}{22}$ of $8) \div (1\frac{1}{2}$ of $2\frac{2}{3}$ of $3\frac{3}{4}$ of $4\frac{4}{5}$ of $5\frac{5}{6})$.

(3.) $\frac{3\frac{2}{3}}{5\frac{7}{9}} \times \frac{3}{8} \times \frac{1}{1\frac{2}{5}} \times \frac{4\frac{7}{8}}{2\frac{3}{5}} \times \frac{11\frac{1}{4}}{7\frac{1}{2}}$ of $\frac{3\frac{2}{3}}{12\frac{1}{10}} \div \frac{1}{8\frac{88}{405}}$; $\frac{7\frac{1}{9} \times \frac{1}{6} \text{ of } \frac{4}{9}}{13\frac{5}{12} - 12\frac{7}{9}}$.

(4.) $\frac{2\frac{3}{4}}{5\frac{7}{9}} \times \frac{1}{8} \times \frac{3}{5} \times \frac{4\frac{3}{8}}{7\frac{1}{2}} \times \frac{7\frac{3}{7}}{5\frac{1}{5}} \times \frac{11\frac{3}{5}}{29}$ of $\frac{4\frac{1}{6}}{13}$ of $\frac{3\frac{1}{4}}{10\frac{3}{8}} \times 31\frac{1}{5}$ of $20\frac{3}{4}$.

(5.) $(\frac{140}{363} \times 1\frac{4}{5} \times \frac{5}{18} \times 1\frac{51}{70} \times \frac{3}{20}) \div (\frac{133}{480} \times \frac{48}{91} \times \frac{143}{19} \times \frac{1}{8} \times \frac{99}{11})$.

(6.) $\frac{6\frac{13}{19} - 4\frac{11}{18}}{\frac{2}{3} \div 15\frac{6}{11}}$; $\frac{8\frac{4}{7}}{3\frac{1}{5}} - \frac{7\frac{5}{9} \text{ of } \frac{3}{17}}{1\frac{2}{11} \text{ of } 3\frac{3}{10}} + \frac{5\frac{5}{8}}{1\frac{11}{24}}$.

(7.) $4\frac{3}{4} + \frac{3}{7}$ of $10\frac{1}{2} + 3\frac{3}{10}$; $\frac{2\frac{1}{2} - \frac{1}{3}}{2\frac{1}{3} - 1\frac{1}{4}}$.

(8.) $3\frac{11}{42} + 7\frac{13}{21} + 9\frac{5}{63} - 19\frac{17}{18}$; $\frac{1}{39}$ of $6\frac{1}{2}$ of $114 - 3\frac{5}{13}$

(9.) $\frac{3}{7} - \frac{7}{9} + \frac{11}{42} + \frac{13}{21} - \frac{15}{126} + \frac{35}{147} + \frac{1}{3}$.

(10.) $\frac{5\frac{3}{4}}{6} + \frac{7\frac{5}{8}}{7} + \frac{3\frac{1}{14}}{5\frac{1}{7}} - \frac{1\frac{5}{26}}{1\frac{1}{13}} - \frac{1\frac{1}{16}}{2\frac{1}{4}}$.

(11.) $\{(\frac{1}{4} - \frac{1}{9}) \times (\frac{3}{5} + 1\frac{1}{2})\} \div (\frac{3}{7}$ of $1\frac{5}{9})$.

(12.) $\frac{2\frac{1}{2} - \frac{5}{6}}{2\frac{1}{2} + \frac{5}{6}} + \frac{7}{12}$ of $\frac{9 \times 10}{14 \times 3} - \frac{22\frac{1}{2}}{30}$.

(13.) $\frac{3}{5}$ of $1\frac{2}{7} + \frac{1}{7}(1 - \frac{2}{7}) + \frac{3}{35}(\frac{1}{7} + \frac{2}{5}) + \frac{2}{25}$; $\frac{6\frac{1}{4} - 1\frac{3}{5}}{1\frac{2}{3} + \frac{11}{12}} - \frac{1\frac{1}{15} \text{ of } 2\frac{5}{8}}{5\frac{1}{5} \div 3\frac{1}{3}}$.

(14.) $(2\frac{2}{5} + 4\frac{1}{15} + 3\frac{1}{3}) \div \left\{(3\frac{1}{3} + 5\frac{1}{5}) \times \frac{5}{64}\right\}$.

(15.) $(5\frac{1}{4} - 4\frac{4}{5} + 3\frac{1}{12}) \div \left(4\frac{1}{4} + 7\frac{1}{7} - \frac{9}{28}\right)$ of $1\frac{3}{25}$.

(16.) $\{(3\frac{3}{10}$ of $7\frac{2}{9}) + (1\frac{5}{6}$ of $\frac{13}{15})\} \div \{(1\frac{3}{8}$ of $13) - (7\frac{1}{3}$ of $1\frac{5}{6})\}$.

(17.) $(7\frac{1}{2} - 1\frac{1}{16} + 5\frac{1}{48}) \div \left(5\frac{1}{12} + 7\frac{1}{5} - \frac{11}{60}\right)$ of $2\frac{3}{11}$.

(18.) $(8\frac{1}{5} - 12\frac{1}{6} + 7\frac{1}{8}) \div (13\frac{1}{12} - 7\frac{1}{4} + 25\frac{3}{4})$.

(19.) $\left(\frac{2\frac{3}{4} + 3\frac{2}{5}}{4\frac{1}{5} + 5\frac{1}{4}} + \frac{3\frac{2}{3}}{10\frac{1}{2}}\right) \div \frac{2\frac{1}{4} - \frac{2}{3} \text{ of } 1\frac{5}{6}}{\frac{1}{5} \text{ of } 3\frac{1}{3} + \frac{13}{36}}$.

(20.) $\frac{2\frac{1}{4} - \frac{2}{3} \text{ of } 1\frac{5}{6}}{\frac{1}{5} \text{ of } 2\frac{1}{2} + \frac{17}{16}} \times \frac{1\frac{3}{4} - \frac{7}{6} \text{ of } \frac{18}{28}}{\frac{5}{6} \text{ of } \frac{12}{20} + 5\frac{1}{2}} \div \left(\frac{\frac{3}{4} + \frac{7}{9} - \frac{5}{12}}{16\,(\frac{3}{8} \text{ of } \frac{2}{9} - \frac{1}{6} \times \frac{3}{8})} \times \frac{1}{3\frac{10}{37}}\right)$

(21.) $\frac{7\frac{5}{9} \times 2\frac{1}{5}}{4\frac{2}{5}\,(\frac{1}{2} - \frac{1}{5})} \div \left(\frac{3\frac{1}{7}}{4 - 3\frac{5}{7}} \times \frac{1\frac{22}{39}}{1\frac{1}{5}} \text{ of } 1\frac{1}{5}\right)$.

(22.) $\dfrac{4\frac{1}{9} - \frac{4}{5} \text{ of } 1\frac{5}{6}}{\frac{1}{5} \text{ of } 4\frac{1}{2} + 1\frac{7}{8}} \times \dfrac{5\frac{1}{4} - \frac{7}{2} \text{ of } \frac{36}{55}}{\frac{5}{2} \text{ of } \frac{6}{10} + 16\frac{1}{2}} \div \dfrac{\frac{3}{8} + \frac{7}{18} - \frac{5}{24}}{8\frac{3}{8} \text{ of } \frac{2}{9} - \frac{1}{3} \text{ of } 3\frac{3}{4}} \text{ of } \dfrac{1}{7\frac{157}{185}}.$

(23.) $\left(\dfrac{3\frac{1}{7}}{4\frac{7}{12}} \text{ of } \dfrac{7\frac{1}{5}}{3\frac{3}{11}} \text{ of } \dfrac{7\frac{1}{24}}{9\frac{1}{63}}\right) \div \left\{ \left(\dfrac{17\frac{1}{9} \text{ of } 3\frac{3}{7}}{2\frac{1}{4}} - \dfrac{11\frac{1}{4} \text{ of } 3\frac{1}{6}}{4\frac{1}{5}}\right) \times \dfrac{13}{6\frac{5}{11}} \text{ of } \dfrac{117}{79} \right\}$

(24.) $\dfrac{\dfrac{3\frac{2}{3}}{5\frac{7}{9}} \times \dfrac{1}{2\frac{2}{3}} \times \dfrac{2\frac{1}{3}}{4\frac{2}{3}} \times \dfrac{4\frac{7}{8}}{2\frac{3}{5}} \times \dfrac{11\frac{1}{4}}{7\frac{1}{2}} \times \dfrac{9\frac{5}{11}}{3\frac{51}{64}} \text{ of } \dfrac{63}{104}}{\dfrac{3}{7} - \dfrac{7}{9} + \dfrac{11}{42} + \dfrac{13}{21} - \dfrac{5}{63} - \dfrac{1}{3}} \text{ of } 1\frac{41}{83}.$

XIX.—REDUCTION OF FRACTIONS, OR FRACTIONAL REDUCTION OF CONCRETE QUANTITIES.

The Fractional Reduction of Concrete Quantities is based on precisely the same principles as reduction of the same in whole numbers.

To proceed from a higher to a lower denomination—**Multiply.**

To proceed from a lower to a higher denomination—**Divide.**

1.—*To find the value of a fractional part of a concrete quantity.*

Rule.—Multiply the concrete number by the fraction; that is, multiply by the numerator, and divide the product by the denominator.

Compound fractions should be previously reduced to simple ones.

Mixed numbers *may be* reduced to improper ones; but it is generally preferable to multiply separately for the whole number, and add to the result the value obtained for the fractional part.

Example I.—Find the value of, (i.) $\frac{5}{9}$ of £3 6s. $4\frac{1}{2}$d.; (ii.) $\frac{2}{15}$ of $\frac{4}{7}$ of £16 8s. $1\frac{1}{2}$d.

$$\begin{array}{lccccccc} & \text{£ s. d.} & & \text{£ s. d.} & & \text{£ s. d} & & \text{£ s. d.} \\ \text{(i.) } \frac{5}{9} \text{ of} & 3\ \ 6\ \ 4\frac{1}{2} & = & \dfrac{3\ \ 6\ \ 4\frac{1}{2} \times 5}{9} & = & \dfrac{16\ \ 11\ \ 10\frac{1}{2}}{9} & = & 1\ \ 16\ \ 10\frac{1}{2}. \end{array}$$

$$\text{(ii.)}\ \frac{2}{\underset{5}{\cancel{15}}} \text{ of } \frac{\cancel{3}}{7} \text{ of } \overset{\pounds}{16}\ \overset{s.}{8}\ \overset{d.}{1\tfrac{1}{2}} = \frac{\overset{\pounds}{16}\ \overset{s.}{8}\ \overset{d.}{1\tfrac{1}{2}} \times 2}{35} = \frac{\overset{\pounds}{32}\ \overset{s.}{16}\ \overset{d.}{3}}{35} = \overset{s.}{18}\ \overset{d.}{9}.$$

Example II.—Find the value of, (i.) $7\frac{3}{4}$ of 5 miles 3 fur. 20 poles; (ii.) $10\frac{5}{12}$ of 3 ac. 3 r. 3 poles.

$$\text{(i.)}\quad \begin{array}{c|c|rrrl} & & \text{miles} & \text{fur.} & \text{po.} & \\ \frac{1}{2} & \frac{1}{2} & 5 & 3 & 20 & \\ & & & & 7\frac{3}{4} & \\ \hline & & 38 & 0 & 20 & \\ \frac{1}{4} & \frac{1}{2} & 2 & 5 & 30 & \rbrace \text{ for } \frac{3}{4} \\ & & 1 & 2 & 35 & \\ \hline & & 42 & 1 & 5 & \end{array}$$

Or thus for $\frac{3}{4}$:—

$$\begin{array}{rrrr} & \text{miles} & \text{fur.} & \text{po.} \\ & 5 & 3 & 20 \\ & & & 3 \\ \hline 4) & 16 & 2 & 20 \\ \hline & 4 & 0 & 25 \end{array}$$

$$\text{(ii.)}\quad \begin{array}{ccrrrl} & & \text{ac.} & \text{r.} & \text{po.} & \\ \frac{1}{12} & \frac{1}{3} & 3 & 3 & 3 & \\ & & & & 10 & \\ \hline & & 37 & 2 & 30 & \\ \frac{4}{12} & \frac{1}{4} & 1 & 1 & 1 & \rbrace \text{ for } \frac{5}{12} \\ & & & 1 & 10\frac{1}{4} & \\ \hline & & 39 & 1 & 1\frac{1}{4} & \end{array}$$

Or thus, for $\frac{5}{12}$:—

$$\begin{array}{rrrr} & \text{ac.} & \text{r.} & \text{po.} \\ & 3 & 3 & 3 \\ & & & 5 \\ \hline 12) & 18 & 3 & 15 \\ \hline & 1 & 2 & 11\frac{1}{4} \end{array}$$

Ex. 52.

Find the value of:—

(1.) $\frac{5}{8}$ of 1s.; $\frac{7}{8}$ of £1; $\frac{5}{18}$ of 4s. 6d.; $\frac{2}{11}$ of 10s. 1d.

(2.) $\frac{1}{8}$ of 5s.; $1\frac{25}{36}$ of 1s. 6d.; $\frac{59}{120}$ of £1.; $\frac{37}{480}$ of £5.

(3.) $\frac{63}{160}$ of £1; $3\frac{9}{10}$ of £1 10s.; $2\frac{14}{25}$ of 10s. $11\frac{1}{4}$d.

(4.) $\frac{7}{22}$ of £4 12s. $1\frac{1}{2}$d.; $\frac{5}{32}$ of $5\frac{1}{3}$ of £273 2s. 6d.

(5.) £2 4s. 2d. $\div\ 1\frac{1}{3}$; £3 14s. 8d. $\div\ 3\frac{5}{8}$; $31\frac{1}{2}$s. $\div\ \dfrac{1\frac{3}{5}}{3\frac{1}{7}}$.

(6.) $\dfrac{15\frac{3}{5}}{7\frac{1}{5}}$ of £1; 184 cwt. 2 qrs. 5 lbs. $\div\ 2\frac{3}{4}$.

(7.) $\frac{7}{8}$ of an acre; $\frac{5}{16}$ cwt; $\frac{4}{5}$ lb. Troy; $\frac{5}{7}$ ton.

(8.) $\frac{3}{10}$ of a day; $\frac{2}{14}$ wk. + $\frac{3}{4}$ day + $\frac{4}{5}$ hr.; 2 wks. 3 d. $\div\ \dfrac{1\frac{1}{5}}{5\frac{1}{3}}$.

(9.) $\frac{1}{14}$ of a guinea — $\frac{1}{15}$ of £1; $\frac{7}{18}$ cwt. + $\frac{4}{7}$ qr. — $\frac{5}{8}$ lb.

(10.) $\frac{5}{8}$ of £2 10s. + $\frac{6}{7}$ of £4 2s. 3d.; $\frac{4}{7}$ cwt. + $8\frac{5}{8}$ lbs. — $3\frac{9}{10}$ oz.

(11.) $\frac{3}{5}$ ac. + $1\frac{5}{8}$ of $1\frac{7}{8}$ ro. + $\frac{4}{7}$ of $8\frac{1}{8}$ po.; $\frac{63}{160}$ ton + $\frac{13}{224}$ cwt.

(12.) $1\frac{1}{6}$ of $3\frac{1}{5}$ fur. — $\frac{3}{14}$ mile; $\frac{5}{8}$ lb. Av. — $\frac{5}{8}$ lb. Troy.

(13.) $\frac{9\frac{7}{9}}{2\frac{1}{27}}$ of a crown + $\frac{18\frac{3}{4}}{5\frac{5}{8}}$ of 27s. — $\frac{1}{3}$ of $5\frac{1}{3}$ of 2s. $9\frac{3}{4}$d.

(14.) $\frac{5}{6} \times \frac{11}{15} \times \frac{9}{10}$ of 10s. — $\frac{2}{7}$ of $\frac{1}{3}$ of £3 11s. 9d.

(15.) ($\frac{3}{4}$ mile + $\frac{3}{22}$ furl. + $\frac{2}{11}$ po.) — ($1\frac{3}{7}$ of $2\frac{5}{8}$ furl. + $2\frac{1}{4}$ of $3\frac{5}{8}$ po. + $\frac{1}{20}$ mile).

Ex. 53.

(1.) Find the value of $\frac{3}{16}$ of a guinea + $1\frac{1}{8}$ of a crown — $\frac{3}{15}$ of half-a-crown.

(2.) Add together $\frac{3}{5}$ of 3s. 4d., $\frac{5}{7}$ of £1 1s. $10\frac{1}{2}$d., and $\frac{9}{11}$ of £2 7s. $2\frac{1}{2}$d.

(3.) Find the value of $\frac{3}{4}$ of 2 guineas + $\frac{4}{7}$ of 15s. 9d. + $\frac{3}{7}$ of 3s. 6d. — $\frac{3}{5}$ of 13s. 4d.

(4.) Add together $\frac{5}{8}$ of a guinea, $\frac{3}{16}$ of a pound, $\frac{7}{10}$ of a crown, and $\frac{5}{6}$ of a shilling.

(5.) Find the sum of $\frac{3}{5}$ of $6\frac{2}{3}$s. + $\frac{5}{7}$ of $43\frac{3}{4}$s. + $\frac{9}{11}$ of £4 14s. 5d. + £$\frac{15\frac{3}{5}}{7\frac{4}{5}}$ — £$3\frac{21}{40}$.

(6.) Add together $\frac{15\frac{3}{5}}{7\frac{4}{5}}$ of £2, $\frac{4}{15}$ of £140 10s. 6d., and $\frac{3}{5}$ of 4s. 2d.

(7.) Add together $\frac{1}{27}$ of £63 9s. $6\frac{3}{4}$d., $\frac{3}{5}$ of $\frac{7}{8}$ of £1, and $\frac{1}{3}$ of $\frac{2}{5}$ of 7s. 6d.

(8.) Find the value of (£17 2s. 9d. ÷ $11\frac{5}{12}$) + (£1 16s. 9d. ÷ $\frac{63}{595}$).

(9.) Find the sum of $\frac{5}{8}$ of a guinea, $\frac{3}{16}$ of a £, $\frac{7}{10}$ of a crown, and $\frac{5}{8}$ of a shilling.

(10.) Find the difference between $\frac{3}{5}$ of 5 guineas, and $\frac{3}{4}$ of $\frac{7}{9}$ of £1.

(11.) Find the value of $\frac{2\frac{1}{9} \div 1\frac{2}{3}}{2\frac{1}{9} - 1\frac{2}{3}}$ of 13s. 4d.; also of $\frac{1\frac{16}{17} - \frac{1}{13} + \frac{3}{34} - \frac{1}{31}}{4\frac{2}{3} \times \frac{3}{7}}$ of 52s.

(12.) Find the value of $\frac{4}{11}$ of 1s. 9d. + $\frac{2}{11}$ of 5s. 3d. — $\frac{2}{11}$ of 8s. 9d.

(13.) Find the value of $12\frac{4}{15}$ of £3 15s. $3\frac{3}{4}$d. — $14\frac{14}{15}$ × £2 6s. $4\frac{3}{4}$d.

(14.) Find the value of $\frac{3}{5}$ of $6\frac{2}{3}$s. + $\frac{5}{7}$ of £37 3s. 9d. + $\frac{9}{11}$ of £4 14s. 5d.

(15.) Divide £85 14s. $3\frac{3}{7}$d. by $109\frac{5}{7}$; and multiply 4s. 7d by $267\frac{3}{16}$.

(16.) Find the value of:—

(i.) $27\frac{3}{4}$ yds. — ($\frac{7}{9}$ of $7\frac{1}{14}$ yds. + $\frac{1}{3}$ of $3\frac{2}{5}$ yds. + $\frac{3}{4}$ yd.).

(ii.) ($1\frac{5}{7}$ of $29\frac{1}{6}$ lbs.) — ($\frac{1}{8}$ qr. + $\frac{5}{112}$ cwt. + $\frac{3}{4}$ of $\frac{7}{11}$ of $9\frac{3}{7}$ lbs.)

(iii.) ($1\frac{5}{7}$ qr. + $\frac{7}{16}$ cwt. + $\frac{1}{8}$ of $\frac{5}{14}$ of a ton) — ($\frac{3}{7}$ cwt. + $8\frac{3}{16}$ lbs. + $3\frac{5}{14}$ qrs.)

(iv.) ($\frac{7}{8}$ bus. + $\frac{3}{4}$ pk. + $\frac{5}{6}$ gal. + $\frac{3}{16}$ qr.) — ($\frac{3}{7}$ bus. + $1\frac{1}{2}$ pk. + $2\frac{2}{3}$ galls.)

2.—*To express one quantity as the fractional part of another.*

Rule.—Reduce both the quantities to the same denomination, and divide the quantity whose fractional value is sought by the other.

This is equivalent to finding what *part* or *multiple* the first quantity is of the second.

Example I.—Express (i.) 5s. $7\frac{1}{2}$d. as a fraction of a guinea; (ii.) £4 10s. $6\frac{3}{4}$d. as a fraction of 11s. $9\frac{3}{4}$d.

$$\text{(i.)}\quad \frac{5\text{s. }7\frac{1}{2}\text{d.}}{21\text{s. }0\text{d.}} = \frac{135}{504} = \frac{15}{56}.$$

Here both sums of money have been reduced to half-pence: now 1 half-penny $= \frac{1}{504}$ of a guinea; therefore 5s. $7\frac{1}{2}$d. or 135 half-pence must $= \frac{135}{504}$ of a guinea.

$$\text{(ii.)}\quad \frac{£4\text{ 10s. }6\frac{3}{4}\text{d.}}{11\text{s. }9\frac{3}{4}\text{d.}} = \frac{4347}{567} = 7\frac{378}{567} = 7\frac{2}{3}.$$

Here both sums have been reduced to farthings; and 1 farthing is $\frac{1}{567}$ of 11s. $9\frac{3}{4}$d.; therefore £4 10s. $6\frac{3}{4}$d., or 4347 farthings must $= \frac{4347}{567}$ of 11s. $9\frac{3}{4}$d.

Example II.—Reduce (i.) 6 yds. 2 ft. to the fraction of a mile; (ii.) $\frac{3}{4}$ of $\frac{6}{7}$ of 3 cwt. 2 qrs. 21 lbs. to the fraction of a ton.

$$\text{(i.)}\quad \frac{6\text{ yds. }2\text{ ft.}}{1760\text{ yds.}} = \frac{20}{5280} = \frac{1}{264}.$$

$$\text{(ii.)}\quad \tfrac{3}{4}\text{ of }\tfrac{6}{7}\text{ of }\frac{3\text{ cwt. }2\text{ qrs. }21\text{ lbs.}}{20\text{ cwt. }0\text{ qrs. }0\text{ lbs.}} = \frac{3}{\underset{2}{\cancel{4}}} \times \frac{\overset{3}{\cancel{6}}}{\cancel{7}} \times \frac{\overset{59}{\cancel{413}}}{2240} = \frac{531}{4480}.$$

Ex. 54.

(1.) Express 9d.; 8d.; $7\frac{1}{2}$d.; $4\frac{3}{4}$d.; $3\frac{1}{2}$d.; $2\frac{1}{4}$d.; $\frac{3}{4}$d. as fractions of 1s.

(2.) Reduce 17s. 6d.; 16s. 8d.; 14s. $6\frac{1}{2}$d.; 13s. 4d.; 1s. 4d.; $8\frac{1}{4}$d.; $3\frac{3}{4}$d. as fractions of £1.

(3.) Express $2\frac{4}{5}$ qrs.; $2\frac{1}{3}$ qrs.; 3 qrs. 2 lbs.; 21 lbs.; $9\frac{15}{17}$ lbs.; as fractions of a cwt.

Express:—

(4.) £3 12s. 4d. as a fraction of £4 1s. 6d.

(5.) £8 16s. $5\frac{1}{2}$d. as a fraction of £1 5s. 8d.

(6.) 3s. $7\frac{1}{2}$d. as a fraction of £2 6s. 9d.
(7.) 13s. 5d. as a fraction of $2\frac{1}{2}$d.
(8.) £1 7s. 6d. as a fraction of £1 10s.
(9.) £2 18s. 2d, as a fraction of £8 4s. 4d.
(10.) £1 17s. $10\frac{1}{2}$d. as a fraction of £5.
(11.) £$1\frac{2}{3}$ as a fraction of $2\frac{1}{3}$ guineas.
(12.) £16 18s. 6d. as a fraction of £10 16s. $4\frac{1}{2}$d.
(13). 3 cwt. 1 qr. 7 lbs. as a fraction of 1 ton.
(14.) 3 cwt. 25 lbs. as a fraction of 1 cwt. 2 qrs. 3 lbs.
(15.) 2 fur. 97 yds. $2\frac{1}{3}$ ft. as a fraction of 1 mile.
(16.) 5 a. $3\frac{5}{8}$ r. as a fraction of 19 a. $2\frac{3}{4}$ r.
(17.) $\frac{7}{12}$ of 2 a. 2 r. 32 p. as a fraction of 2 a. 1 r. 18 p.
(18.) 3 weeks 6 days 16 hrs. as a fraction of 4 weeks.
(19.) Reduce £1 5s. $6\frac{1}{5}$d. to the fraction of £3 2s. $4\frac{3}{5}$d.
(20.) Reduce 2 qrs. 6 lbs. 3 oz. $8\frac{3}{5}$ dr. to the fraction of a cwt.
(21.) What fraction of £7 16s. 3d. is $\frac{6}{11}$ of £7 3s. $2\frac{3}{4}$d?
(22.) How often is $\frac{3}{5}$ of £1 11s. 8d. contained in 19 guineas?

(23.) Express the difference between $\frac{1}{3}$ of 18s. 4d., and $\frac{3}{5}$ of a crown, as fractions of 2s. 6d.

(24.) Reduce $\frac{3}{10}$ of £2 0s. $3\frac{3}{4}$d. to the fraction of £1 4s. $2\frac{1}{4}$d. and 2 qrs. 3 lbs. 1 oz. to the fraction of 1 cwt. 3 qrs. 0 lbs. 14 oz.

(25.) What fraction of a guinea is $\frac{1}{3}$ of $\frac{6}{11}$ of half-crown; and what fraction of half-crown is $\frac{3}{5}$ of 6s. 8d.?

(26.) Reduce $2\frac{2}{3}$ of 3 r. 6 p. to the fraction of 1 a. 2 r. 8 p; $\frac{3}{5}$ of $1\frac{1}{2}$ of 10s. $7\frac{1}{2}$d. to the fraction of £4 4s. $4\frac{1}{2}$d.

(27.) Reduce $\frac{3\frac{3}{8}}{3\frac{5}{8}}$ ($\frac{19}{120}$ of £1—$\frac{7}{48}$ of 1s.) to the fraction of 27s.

(28.) Express $3\frac{2}{7}$ weeks as the fraction of $2\frac{1}{3}$ days; and $\frac{10}{11}$ of $\frac{3}{4}$ of a day as fractions respectively of $4\frac{1}{3}$ hours and of $1\frac{5}{9}$ weeks.

(29.) Reduce $\frac{3}{5}$ of £3 7s. 6d. + $\frac{3}{5}$ of $1\frac{1}{4}$ of $4\frac{1}{2}$ guineas to the fraction of £4.

(30.) What part of half-a-crown is $\frac{1}{7}$ of $\frac{2\frac{1}{4}—(\frac{2}{3} \text{ of } 1\frac{3}{5})}{(\frac{1}{5} \times 3\frac{1}{3}) + \frac{13}{36}}$ of $\frac{2}{11}$ of a guinea.

Ex. 55.

Miscellaneous Examples in Vulgar Fractions.

(1.) Find the G. C. M. of 17255 and 14161; and the L. C. M. of 40, 42, 44, 48.

(2.) Find the G. C. M. of 18, 96, 120, and 8420; also the L. C. M. of 120, 140, 168, and 210.

(3.) Find the Least Common Multiple of 15, 17, 34, 45; and reduce $\frac{11880}{39528}$ to its lowest terms.

(4.) Find the greatest and the least of the fractions:—(i.) $\frac{3}{4}$, $\frac{7}{12}$, $\frac{2}{3}$, $\frac{1}{6}$, and $\frac{1}{2}$; (ii.) $\frac{11}{12}$, $\frac{29}{30}$, $\frac{17}{18}$, $\frac{7}{16}$, and $\frac{47}{48}$.

(5.) Find the Least Common Multiple of 15, 36, 40, 20, and also of 2, 14, 16, 18, 30, 40, and find the Greatest Common Measure of the two results.

(6.) A number between 7000 and 8000 is exactly divisible by each of the numbers 5, 7, 15, 24, 27. Find this number.

(7.) Add together $2\frac{7}{8}$, ($\frac{3}{8}$ of $9\frac{3}{5}$) and $3\frac{11}{12}$. From 114 take $7\frac{5}{19}$ of $12\frac{3}{8}$.

(8.) Simplify (i.) $(2\frac{1}{2} \div 3\frac{1}{4}) + (4\frac{5}{13} \div 2\frac{4}{7})$; (ii.) $13\frac{8}{15} - 5\frac{5}{27} - 6\frac{3}{45} + 5\frac{1}{12}$.

(9.) Multiply $2\frac{1}{4}$ of $10\frac{2}{3}$ of 15 by $7\frac{1}{3}$ of $\frac{1}{141}$; also divide $7\frac{1}{6}$ by $2\frac{4}{7}$.

(10.) What fraction of £7 16s. 3d. is $\frac{6}{11}$ of £7 3s. $2\frac{3}{4}$d.?

(11.) Reduce to their lowest terms: (i.) $\frac{11060}{35581}$; (ii.) $\frac{12818}{22185}$; (iii.) $\frac{21160}{32395}$; (iv.) $\frac{285714}{857142}$; (v.) $\frac{18455}{11385}$.

(12.) Is $\frac{13}{77}$ greater or less than $\frac{14\frac{1}{3}}{88}$, and by how much?

(13.) Find the value of $\frac{3}{4}$ of a guinea + $\frac{3}{5}$ of a crown — $\frac{3}{4}$ of 2d.

(14.) From 3 tons $13\frac{3}{5}$ cwt. take 2 tons $9\frac{15}{27}$ cwt.

(15.) How many times can a vessel containing $1\frac{3}{8}$ pints be filled from one containing $27\frac{3}{5}$ pints?

(16.) Find the Greatest Common Measure of 2929 and 2291. Also of 2929, 2291, and 2871.

(17.) Find the product of the sum and difference of the greatest and the least of the fractions $\frac{31}{60}$, $\frac{13}{25}$, and $\frac{5}{9}$.

(18.) Reduce the following expressions to their simplest forms:—

(i.) $7\frac{1}{2} + \frac{5}{6} + \frac{3}{8}$ of $\frac{3}{4}$ of $19\frac{5}{6}$. (ii.) $\frac{4}{5} + \frac{1}{8} - \frac{2}{13} + \frac{17}{19} - \frac{11}{12}$.

(iii.) $\left(\frac{1}{3\frac{1}{5}} - \frac{2\frac{1}{4}}{9} + \frac{3\frac{5}{8}}{2}\right) \div (3\frac{2}{5} \times 2\frac{1}{4})$. (iv.) $\frac{(\frac{4}{5} + \frac{3}{7}) + \frac{1}{6} \text{ of } \frac{2}{5}}{\frac{3}{5} + \frac{7}{12} - \frac{1}{8}}$

(v.) $\frac{4\frac{1}{7} - 2\frac{1}{4}}{6\frac{1}{2} - 2\frac{1}{7}}$ (vi.) $\frac{\frac{5}{9} \text{ of } \frac{7}{8} \text{ of } \frac{6}{15}}{\frac{3}{4} + 1\frac{5}{7} - \frac{18}{10}}$ (vii.) $\frac{4\frac{3}{10} \text{ of } 2\frac{1}{6}}{7\frac{2}{9} - 3\frac{1}{5}}$

(viii.) $(\frac{8}{9} \times \frac{5}{7}) + (\frac{4}{15} \times \frac{1}{10}) + \frac{3}{5}(\frac{1}{12} + \frac{11}{14}) \div \frac{3}{70}(\frac{4}{7} - \frac{4}{15})$.

(19.) How many times are £$12\frac{3}{10}$ contained in £$236\frac{3}{12}$?

(20.) Divide $2\frac{1}{5}$ of $3\frac{1}{4}$ by $5\frac{1}{2}$ of $4\frac{1}{3}$; also $13\frac{3}{14}$ by $\frac{1}{\frac{3}{5} \text{ of } 2\frac{1}{3}}$

(21.) Find the Greatest Common Measure of: (i.) 28, 84, 154, and 343; (ii.) 396, 5184, and 6914.

(22.) Write down all the prime numbers under 50.

(23.) Find the factors, G. C. M., and L. C. M. of 126 and 144.

(24.) Compare the values of $\frac{2\frac{1}{3}}{27}$, $\frac{8}{81}$, and $\frac{20}{243}$; also of $\frac{2\frac{1}{3}}{3\frac{1}{3}}$, $\frac{18\frac{1}{5}}{21}$, and $\frac{17\frac{1}{3}}{29\frac{5}{7}}$.

(25.) From $2\frac{1}{3}$ of $4\frac{1}{10}$ take $2\frac{1}{3} + 4\frac{1}{10}$. Subtract $\frac{5}{6} + \frac{7}{12} + \frac{3}{4}$ from $\frac{2}{3} + \frac{3}{4} + \frac{11}{12}$.

(26.) Simplify:—

(i.) $10\frac{1}{3} \times 3\frac{1}{7} - (4\frac{3}{5} \div 3\frac{1}{3})$; (ii.) $\{(10\frac{1}{3} \times 3\frac{1}{7}) - 4\frac{3}{5}\} \div 3\frac{1}{3}$;

(iii.) $(6\frac{1}{15} - 4\frac{3}{20} \div 1\frac{9}{11} \times 1\frac{1}{10} \div 1\frac{1}{5})$;

(iv.) $(5\frac{1}{4} + 3\frac{9}{11}) \div (\frac{3}{7} \div 1\frac{1}{7})$; (v.) $5\frac{1}{4} + \{3\frac{9}{11} \div (\frac{3}{7} \div 1\frac{1}{7})\}$;

(vi.) $\frac{3\frac{1}{2} - 2\frac{1}{6}}{\frac{1}{4} \text{ of } (\frac{1}{5} + \frac{1}{7})} \div 15\frac{5}{9}$; (vii.) $\frac{7}{8}$ of $\frac{3\frac{2}{5} + 2\frac{3}{4}}{5\frac{1}{4} + 4\frac{1}{5}} + \frac{1\frac{5}{6}}{5\frac{1}{4}}$;

(viii.) $\frac{15}{17}$ of $\frac{13\frac{3}{5}}{5\frac{5}{11}}$ of $(5\frac{5}{9} \div 7\frac{7}{9})$.

(27.) Find the least common multiple of:—

(i.) 24, 10, 32, 45, and 25. (ii.) 225, 255, 289, 1023, and 4095.

(28.) If a rupee be worth 2s. $0\frac{1}{2}$d., and a dollar 4s. $4\frac{1}{2}$d., find the least number of rupees which make an exact number of dollars.

(29.) Standard gold being coined at the rate of £3 17s. $10\frac{1}{2}$d. per oz., what is the least number of ounces that can be coined into an exact number of sovereigns?

(30.) What number must be added to $\frac{2\frac{1}{3} + 5\frac{6}{7} - 6\frac{1}{2}}{16\frac{2}{3} - 10\frac{9}{10} - 5\frac{5}{7}}$ to make 45?

(31.) Reduce $\frac{3\frac{1}{3}}{1\frac{1}{15}}$ of $\{\frac{19}{120}$ of £3 — $\frac{7}{48}$ of 1s.$\}$ to the fraction of £3 18s.

(32.) If a train runs $\frac{8}{15}$ of a mile per minute (i.) what time will it take to travel $\frac{2}{3}$ of $\frac{4}{7}$ of $89\frac{1}{4}$ miles; and (ii.) how far will it have gone in $\frac{2}{3}$ of 3 hrs. 27 min.

(33.) If A can do a piece of work in 5 days, which B can do in 8 days; how many days will they take, both working together?

(34.) If A. can mow $\frac{2}{3}$ of a field in $1\frac{1}{3}$ days, and B. $\frac{3}{10}$ of it in $2\frac{1}{4}$ days, in what time can they mow it, both working together?

(35.) A cistern can be filled by one pipe in $21\frac{3}{5}$ minutes, and emptied by another in $28\frac{2}{3}$ minutes, how long will it take to fill when both pipes are left open?

(36.) Find the difference in weight between a sovereign and a napoleon; when 623 sovereigns weigh $\frac{25}{36}$ of $19\frac{1}{5}$ lbs. troy, and 155 napoleons weigh 2 lbs. $8\frac{37}{240}$ oz. troy.

XX.—DECIMAL FRACTIONS.

The *actual* value of any figure in a line of figures depends on the place which it occupies in the line, each figure increasing *tenfold* in value for every place it occupies *to the left* of the units' place.

Thus, 6789 = 6 thousands + 7 hundreds + 8 tens + 9 units.

If this method of notation be extended to figures standing to the right of the units' place, we shall be able to express numbers less than unity, as, *tenths*, *hundredths*, *thousandths*, &c., decreasing in a tenfold ratio the further they are removed from the units' place. These numbers are called **Decimal Fractions**, or simply **Decimals.**

The units' figure having now other figures *both* to the right and left, it will be necessary to distinguish it by some sign or other. The sign generally used is a dot, called the **decimal point**, placed to the right of the units' place.

Thus $5.6789 = 5 + \frac{6}{10} + \frac{7}{100} + \frac{8}{1000} + \frac{9}{10000} = 5\frac{6789}{10000}$.

The number of figures to the right of the decimal point is called the number of decimal places; thus, in 5.6789, there are *four* decimal places.

A **Decimal Fraction,** then, is a fraction whose denominator is 10, or some power of 10, as 100, 1000, 10000, &c.

Thus $.5 = \frac{5}{10}$; $.75 = \frac{75}{100}$; $.875 = \frac{875}{1000}$.

When a decimal is expressed as a vulgar fraction, the number of ciphers, or noughts, in the denominator is equa to the number of decimal places.

Ciphers placed to the right of a decimal do not alter i value.

Thus .5, .50, .500 are of equal value, for

$$.500 = \frac{5\cancel{00}}{10\cancel{00}};\ .50 = \frac{5\cancel{0}}{10\cancel{0}};\ .5 = \frac{5}{10}.$$

A decimal is **multiplied** by 10, 100, 1000, &c., 10^m, by removing the decimal point 1, 2, 3, &c., m, places to the *right.*

Thus $.437 \times 10 = 4.37$; for $\frac{437}{100} \times \frac{1\cancel{0}}{1} = \frac{437}{10\cancel{0}} = 4\frac{37}{100} = 4.37$;

Also $7.6314 \times 1000 = 7631.4$; for $7\frac{6314}{10000} = \frac{76314}{10000}$

and $\frac{76314}{10\cancel{000}} \times \frac{1\cancel{000}}{1} = \frac{76314}{10} = 7631\frac{4}{10} = 7631.4$.

A decimal is **divided** by 10, 100, 1000, &c., 10^m, by removing the decimal point 1, 2, 3, &c., m places to the *left.*

Thus $45.69 \div 10 = 4.569$; for $45\frac{69}{100} \div 10 = \frac{4569}{100} \times \frac{1}{10} = \frac{4569}{1000} = 4\frac{569}{1000} = 4.569$.

Ciphers may be added to the left of decimals, after the decimal point: they are needed, as in whole numbers, to fill up vacant places, so that the succeeding figures may have their true local value.

One or more ciphers thus prefixed to the left of a decimal, have the effect of dividing it by 10, 100, &c., according to their number.

Thus $.7 \div 100 = .007$; for $\frac{7}{10} \div 100 = \frac{7}{10} \times \frac{1}{100} = \frac{7}{1000} = .007$.
Also $45.69 \div 10000 = .004569$; for $45\frac{69}{100} \div 10000 = \frac{4569}{100} \times \frac{1}{10000} = \frac{4569}{1000000} = .004569$.

A decimal may at once be written as a vulgar fraction by writing the significant figures for the numerator, and, for the denominator, 1, followed by as many ciphers as there are decimal places.

Thus $.7 = \frac{7}{10}$; $.002 = \frac{2}{1000}$; $4.0175 = 4\frac{175}{10000}$.

Ex. 56.

Express the following vulgar fractions in the decimal form:—

(1.) $\frac{3}{10}, \frac{5}{100}, \frac{7}{1000}, \frac{9}{10000}$. (2.) $\frac{27}{100}, \frac{319}{1000}, \frac{47}{1000000}$.
(3.) $\frac{3287}{100}, \frac{4681}{1000}, \frac{976}{100000}$. (4.) $\frac{4374}{10000}, \frac{397}{10}, \frac{86532}{100}$.

ress the following decimals in the form of vulgar fractions :—

) .6, .19, .134, .8967. (6.) .07, .0002, .01007.

) .000009, 761.27, .0017. (8.) 4.11, 15.017, 271.6032.

) .00739, .15625, 870.001, 27.000007, 178.002.

tiply and divide :—

) 43.567 by 10 ; .6576 by 100 ; .0674 by 10000.

) 384.0001 by 1000 ; .005 by 1000000 ; 873.65 by 100.

) 1376.4 by 10000 ; 1.0003 by 100 ; .00076 by 1000.

XI.—ADDITION AND SUBTRACTION OF DECIMALS.

Addition and **Subtraction** of **Decimals**, as in numbers, figures having the same local value must ded or subtracted.

nits are placed under units, as in whole numbers, the decimal which stand to the right of the units' place, will be under each and tenths will be under tenths, hundredths under hundredths, on.

e, when thus arranged, all the figures in any one column are ators of fractions, having a common denominator; they may be or subtracted : hence the following rule :—

LE.—Place the numbers so that the decimal points be under each other. Add or subtract as in whole ers, placing the decimal point in the result under the points.

MPLE.—(i.) Add together 26.151, 2.3584, 76, 123.462, 53.84 . (ii.) Find the difference between 145.001 and 17.999 ; (iii.) also n .8265 and .818365.

Addition.	*Subtraction.*
(i.) 26.151	(ii.) 145.001
2.3584	17.999
76.	127.002
123.462	
53.84	(iii.) .8265
7.3815	.818365
289.1929	.008135

Ciphers *may* be added to fill up the vacant places *to the right* of the decimals, but simply for the convenience of guiding the eye.

Ex. 57.

Add together:—

(1.) 4.13295, 42.171, .3721, .0064, .6, 25.
(2.) 3010.3, 21.747, .006887, .96, 325.37, 141.2.
(3.) .0065, 65, .065, 6.5, .000065, .65.
(4.) 16.5, 176.1105, 11.0012, 18.75, 11.5, 14.123.
(5.) .813, 72.5, 32.071, 2.1574, 371.4, 2.75.
(6.) 7.1, 3275, 3.2675, 1 00005, 14.7, .001.

Subtract:—

(7.) 215.7 from 271.
(8.) 31.765 from 45.31.
(9.) .001478 from 378.62.
(10.) .300303 from 300.30101.
(11.) 2.0900683 from 9.79076.
(12.) 47.1578 from 63.00706.

(13.) 91.75 + 543.175 — 89.005 — 17.007 + 161.1003 — 317.
(14.) 17.48 — 3.586 + .374 + .00076 — 41.9 + 157.01.
(15.) .0078 — .172 — .78154 + 37.011 — 11.9901 + 169.999.
(16.) 51.16 + .0073 — 89.4 — 3.90909 + 41.051 — 2.09091.

Metric System.

The **Metric System** of Weights and Measures, the **French** coinage and the **Decimal coinage**, proposed for the United Kingdom, being all decimal systems, questions in which they occur are solved by the rules of Decimal Fractions.

EXAMPLE.—(i.) Add together 200 francs 79 centimes, 15 fr. 60 c., 97 fr. 18 c., 371 fr. 07 c. (ii.) From 978 hectolitres 5 litres take 259 hectolitres 17 litres 25 centilitres.

```
      fr. c.                     hectolit. lit.
 (i.) 200.79               (ii.) 978,05
       15.60                     259,17.25
       97.18                     ---------
      371.07                     718,87.75 =
      ------             718 hectolit. 87 lit. 75 centilit.
      684.64 =
684 francs 64 centimes.
```

Ex. 58.

Add together:—

(1.) 760 fr. 25 c., 401 fr. 13 c., 967 fr. 75 c., 43 fr. 8 c., 62 fr. 5 c.

(2.) 594 fr. 30 c., 4000 fr., 131 fr. 5 c., 253 fr. 15 c., 673 fr. 43 c.

(3.) 35 hectolit. 2 lit. 80 centilit., 37 hectolit. 5 lit., 61 hectolit. 14 lit. 17 centilit.

(4.) 37 kilog. 56 gram. 71 centig., 5 kilog. 9 hectogr. 15 gram. 5 decig., 25 decagr. 27 centigr., 4 kilog. 57 gram. 7 decigr.

(5.) From 30 fr. 5 c. take 15 fr. 87 c.; 48 fr. 18 c. — 37 fr. 59 c.

(6.) 178 hectolit. 17 lit. — 37 hectolit. 29 lit. 8 decilit; 284 kilog. 47 gr. — 97 kilog. 5 decagr. 37 centigr.

(7.) 424 hectares 17 ares + 1000 hectar. 72 ar. 9 centiar. + 69 hectar. 5 ar. 17 centiar.

(8.) 11 kilomet. 17 met. 5 centim. + 37 kilom. 6 met. 25 centim. — 9 kilom. 3 met. 75 centim.

XXII.—MULTIPLICATION OF DECIMALS.

Rule.—Multiply as in whole numbers, and point off in the product as many decimal places *as there are in the multiplier and multiplicand together.*

Example.—Multiply (i.) 87.7 by 2.03; also (ii.) .0071 by .012.

$$\text{(i.)}\quad \begin{array}{r} 87.7 \\ 2.03 \\ \hline 2.631 \\ 175.4 \\ \hline 178.031 \end{array} \qquad\qquad \text{(ii.)}\quad \begin{array}{r} .0071 \\ .012 \\ \hline .0000852 \end{array}$$

These may be *proved* by Vulgar Fractions, thus:

$$\text{(i.) } 87.7 \times 2.03 = \frac{877}{10} \times \frac{203}{100} = \frac{178031}{1000} = 178.031.$$

Now the number of ciphers in the denominator of the product is equal to the sum of the number of ciphers in the denominator of each factor, and the number of ciphers is equal to the number of decimal places; therefore the number of decimal places in the product must be equal to the *sum* of the number of decimal places in each factor.

$$\text{(ii.) } .0071 \times .012 = \frac{71}{10000} \times \frac{12}{1000} = \frac{852}{10000000} = .0000852.$$

Here, since there are *seven* ciphers in the denominator, *seven* decimal places must be marked off; but as there are but three significant figures, four ciphers must be prefixed to make up the required number.

Ex. 59.

(1.) 30.5 × 7.5. (2.) 25.6 × .25. (3.) .59 × 83.
(4.) 47.001 × .304. (5.) 140.03 × .007. (6.) 8.7651 × 30.
(7.) .769 × 2900. (8.) .0029 × 2.51. (9.) 732 × .625.
(10.) 100.001 × 100.1. (11.) 1.011 × 10.11. (12.) .0025 × 1210.
(13.) 1.01 × 10.1 × 101 × .101. (14.) .05 × .005 × .5 × 500.
(15.) 30.92 × 2.07 × .51. (16.) 10.3 × .011 × 15.99.

XXIII.—DIVISION OF DECIMALS.

Rule.—Divide as in whole numbers, and point off in the quotient as many decimal places *as the number of decimals in the dividend exceeds the number of decimals in the divisor.*

If the number of decimal places in the dividend is *equal* to the number in the divisor, *none* will have *to be pointed off*, and the quotient will be a *whole number.*

If the number of decimal places in the dividend is *less* than the number in the divisor, *add ciphers to the dividend* till the number of decimals in each is the same. The quotient resulting from the division up to the last cipher so added, will be a whole number.

Since ciphers added to the right of a decimal have no effect on its value, as many ciphers may be so added, as may be found necessary for continuing the division; *ciphers thus added must be counted as decimal places,* in determining the value of the quotient.

Example I.—Divide (i.) 114.375 by 15.25; (ii.) 3.2 by 12.8.

```
(i.) 15.25)114.375(7.5
           106.75
           ------
             7 625
             7 625
             -----
             . ...
```

The *number* of decimals in the dividend is 3, and in the divisor 2; therefore point off 1 in the quotient. 3 — 2 = 1.

```
(ii.) 12.8)3.20(.25
           2 56
           ----
            640
            640
            ---
            ...
```

The number of decimals in the dividend, including the added ciphers, is 3, and in the divisor 1; therefore point off 2 in the quotient.

Example II.—Divide (iii.) 114.375 by 1.525; (iv.) .32 by 128.

$$\text{(iii.)}\quad \begin{array}{r} 1.525)\,114.375\,(75 \\ \underline{106\ 75} \\ 7\ 625 \\ \underline{7\ 625} \\ \cdots\cdot \end{array}$$

The number of decimals in the dividend and divisor being the same, the quotient is a whole number.

$$\text{(iv.)}\quad \begin{array}{r} 128)\,.320\,(.0025 \\ \underline{256} \\ 640 \\ \underline{640} \\ \cdots \end{array}$$

The number of decimals in the dividend, including the added ciphers, is 4, while there are no decimals in the divisor; therefore 4 must be pointed off in the quotient.

The preceding examples may be thus *proved* by Vulgar Fractions:—

$$\text{(i.)}\quad 114.375 \div 15.25 = \frac{\overset{75}{\cancel{114375}}}{10\cancel{00}} \times \frac{1\cancel{00}}{\cancel{1525}} = \frac{75}{10} = 7.5.$$

$$\text{(ii.)}\quad 3.2 \div 12.8 = \frac{\cancel{32}}{\cancel{10}} \times \frac{\cancel{10}}{\underset{4}{\cancel{128}}} = \frac{1}{4} = \frac{25}{100} = .25.$$

$$\text{(iii.)}\quad 114.375 \div 1.525 = \frac{114375}{\cancel{1000}} = \frac{\cancel{1000}}{1525} = \frac{114375}{1525} = 75.$$

$$\text{(iv.)}\quad .32 \div 128 = \frac{\cancel{32}}{100} \times \frac{1}{\underset{4}{\cancel{128}}} = \frac{1}{400} \quad \frac{25}{10000} = .0025.$$

The use of a little common sense will generally enable one to see whether the quotient is correctly pointed: thus, in (i.), 15 is contained in 114 *seven* times with a remainder, not 70 times; and in (ii.), 3 is a fractional part of 12, therefore there can be no whole number in the quotient.

The true local value of any figure in the quotient may at all times be ascertained without counting by dividing the local value of any figure of the dividend by the local value of that figure of the divisor whose multiple, by the quotient figure, stands beneath it in the subtrahend; thus, in (iii.), the 7 in the dividend represents 7 *hundredths*, and the 5 below it arises from the multiplication of the 5 in the divisor, the local value of which is 5 *thousandths*.

Now $\frac{1}{100} \div \frac{1}{1000} = \frac{1}{1\cancel{00}} \times \frac{10\cancel{00}}{1} = 10$, therefore the local value of the 7 in the quotient must be 7 *tens*, or 70.

Sometimes the process of division does not terminate, but the same figure or figures will perpetually *recur*.

Decimals which thus recur are called **Circulating, Repeating, or Recurring Decimals.**

A **Recurring Decimal** is marked by a dot over the first and last figures of the part that recurs; thus, $.\dot{5}$, $.\dot{3}\dot{6}$, $.\dot{1}4285\dot{7}$.

Ex. 60.

(1.) 228.75 ÷ 7.5.
(2.) 3.1416 ÷ .7854.
(3.) 1.96042 ÷ 140.03.
(4.) 2230.1 ÷ .769.
(5.) .3456 ÷ .0054.
(6.) .00456 ÷ .1368.
(7.) 12.55 ÷ .01004.
(8.) 1255 ÷ 10.04.
(9.) .001255 ÷ 1004.
(10.) 3.1146 ÷ .036.
(11.) 2.568 ÷ 15.
(12.) .0495 ÷ .03888.
(13.) 7.1125 ÷ 6.5.
(14.) 6.1325 ÷ .175.
(15.) .0625 ÷ 1.75.
(16.) 4.8 ÷ .0032.
(17.) 3.1 ÷ .0125.
(18.) .0077 ÷ .875.

Simplify :—

(19.) (4.5306 + .0007 + .0318) ÷ (3.015 + 11.0005 + 8.8).

(20.) (3.5 — .04) ÷ (5 — .0625).

(21.) (.2453 × .0425) ÷ (.011 + .4125 — .1782).

(22.) (.00441 + .351) ÷ (.0072 — .00072).

(23.) $$\frac{.0016 \times .025}{.325 \times .05} \div \frac{.1216 \times .105 \times .002}{.08512 \times .625 \times .039}.$$

(24.) $$\left(\frac{12.5 \div .0025}{1.05 + 2.005 + 1.945}\right) \times \left(\frac{1 - .05}{5 + .5} \times \frac{3 - .8}{38}\right).$$

To express a Vulgar Fraction in the form of a Decimal.

Since a Vulgar Fraction may be considered as expressing the division of the numerator by the denominator, we have but to perform that division decimally; hence the following:—

RULE.—Divide the numerator by the denominator, annexing to the right of the numerator as many ciphers as may be found necessary, separating them from the numerator by a point.

The number of decimal places will be equal to the number of ciphers annexed.

Example.—Express $\frac{1}{25}$, $\frac{7}{16}$, $\frac{15}{128}$, $\frac{104}{111}$, as decimals.

$\frac{1}{25} = 1.00 \div 25 = .04$; $\frac{7}{16} = 7.0 \div 16 = .4375$.

$\frac{15}{128} = 15.0 \div 128 = .1171875$; $\frac{104}{111} = 104.0 \div 111 = .\dot{9}3\dot{6}$.

If the Vulgar Fraction has for its denominator 10, or some power of 10, it has been already shown [Page 103] that it may be at once expressed decimally by pointing off from the right hand of the numerator as many figures as there are ciphers in the denominator.

Thus $\frac{7}{10} = .7$; $\frac{183}{100} = 1.83$; $\frac{3}{10000} = .0003$.

Ex. 61.

Express as decimals :—

(1.) $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{8}$. (2.) $\frac{5}{8}$, $\frac{7}{8}$, $\frac{1}{16}$, $\frac{9}{16}$.

(3.) $\frac{15}{64}$, $\frac{17}{128}$, $\frac{21}{250}$, $\frac{3}{40}$. (4.) $\frac{13}{25}$, $\frac{11}{32}$, $\frac{7}{125}$, $\frac{1}{256}$.

(5.) $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{9}$, $\frac{5}{9}$. (6.) $\frac{4}{7}$, $\frac{6}{7}$, $\frac{5}{14}$, $\frac{13}{28}$.

(7.) $\frac{7}{32}$, $\frac{13}{27}$, $\frac{11}{15}$, $\frac{5}{6}$. (8.) $\frac{23}{33}$, $\frac{1}{13}$, $\frac{1}{148}$, $\frac{11}{21}$.

To express a Decimal in the form of a Vulgar Fraction.

1. If the Decimal be **finite**, or *terminate*, set down the significant figures which compose it for a numerator, and for a denominator, 1, followed by as many ciphers as there are decimal places [page 104].

Thus $.625 = \frac{625}{1000} = \frac{25}{40} = \frac{5}{8}$; $.036 = \frac{36}{1000} = \frac{9}{250}$.

2. If the Decimal be **recurring**, or *interminate*, set down for a **numerator** *all* the significant figures to the end of the first period, *subtracting the figures (if any) which do not recur.*

For the **denominator**, set down as many *nines* as there are recurring figures, followed by as many *ciphers* as there are non-recurring figures.

Example.—Express $.\dot{3}$, $.\dot{6}7\dot{5}$, $.0\dot{6}$, $.24\dot{5}\dot{7}$, as Vulgar Fractions.

$$.\dot{3} = \frac{3}{9} = \frac{1}{3}; \quad .\dot{6}7\dot{5} = \frac{675}{999} = \frac{75}{111} = \frac{25}{37}.$$

$$.0\dot{6} = \frac{6}{90} = \frac{1}{15}; \quad .24\dot{5}\dot{7} = \frac{2457-24}{9900} = \frac{2433}{9900} = \frac{811}{3300}.$$

The reason for the rule may be shown as follows:—

Suppose it is required to express $.\dot{6}$, $.\dot{9}6\dot{3}$, $.75\dot{5}6\dot{7}$ as vulgar fractions; the first object will be to get rid of the recurring part beyond the first period. To do this,

First. Multiply the given decimal by its denominator, that is by 1 followed by as many ciphers as there are decimal places to the end of the first period;

Secondly. Multiply the given decimal by the denominator of the non-recurring part, that is by 1 followed by as many ciphers as there are non-recurring figures;

Thirdly. Subtract the second product from the first, and

Fourthly. Divide the remainder, which will always be a whole number, by the difference between the two multipliers.

The quotient will be the value of the decimal expressed as a vulgar fraction.

$$
\begin{array}{llcl|llcl}
.\dot{6} & \times & 10 = & 6.\dot{6} & .\dot{9}6\dot{3} & \times & 1000 = & 963.\dot{9}6\dot{3} \\
.\dot{6} & \times & 1 = & .\dot{6} & .\dot{9}6\dot{3} & \times & 1 = & .\dot{9}6\dot{3} \\
\hline
.\dot{6} & \times & 9 = & 6 & .\dot{9}6\dot{3} & \times & 999 = & 963 \\
.\dot{6} & & = & \frac{6}{9} = \frac{2}{3} & .\dot{9}6\dot{3} & & = & \frac{963}{999} = \frac{107}{111}
\end{array}
$$

$$
\begin{array}{llcl}
.75\dot{5}6\dot{7} & \times & 100000 = & 75567.\dot{5}6\dot{7} \\
.75\dot{5}6\dot{7} & \times & 100 = & 75\,567 \\
\hline
.75\dot{5}6\dot{7} & \times & 99900 = & 75492 \\
.75\dot{5}6\dot{7} & & = & \frac{75492}{99900} = \frac{699}{925}.
\end{array}
$$

Ex. 62,

Express the following either as vulgar fractions or mixed numbers—

(1.) $.\dot{4}$, $.\dot{3}\dot{6}$, $1\dot{4}\dot{4}$, $.0\dot{7}\dot{5}$.

(2.) $3.4\dot{1}\dot{8}$, $1.1\dot{4}5\dot{9}$, $13.2\dot{0}\dot{6}$.

(3.) $41.7\dot{5}4\dot{3}$, $.70\dot{4}\dot{5}$, $.00\dot{8}7\dot{9}$.

(4.) $.\dot{0}2\dot{7}$, $.0\dot{2}\dot{7}$, $.02\dot{7}$, $.2\dot{7}\dot{0}$.

(5.) $.\dot{1}4285\dot{7}$, $.\dot{4}2857\dot{1}$, $9.92\dot{9}$.

(6.) $12.09\dot{6}59\dot{7}$, $.00040\dot{6}$, $.12\dot{3}\dot{6}$.

(7.) $.20\dot{2}3809\dot{5}$, $.5\dot{4}95\dot{0}$.

(8.) $130.\dot{8}5714\dot{2}$, $.078\dot{7}0\dot{3}$.

XXIV.—RECURRING DECIMALS.

Recurring Decimals may be **added or subtracted** with sufficient exactness for all practical purposes, by extending the periods to two or three places more than the number of decimals required to be accurate, and then adding or subtracting as the case may be.

Example.—Find the value (correct to 6 places of decimals):

(i.) of $11.7\dot{4} + 1.\dot{0}3\dot{2} + 15.4\dot{5}1\dot{6} + .00\dot{7}\dot{8} + 329\,\dot{1}45\dot{6} + 25.\dot{8}$;

(ii.) of $28.4\dot{3}0\dot{7} - 15\,90\dot{2}37\dot{8}$.

(i.) 11.74444444
1.03203203
15.45165165
.00787878
329.14561456
25.88888888
383.27051035

(ii.) 28.43073073
15.90237823
12.52835249

In these examples the decimals have been carried out to *eight* places, that is, two more than the number required to be accurate.

For *strict accuracy* they should have been carried out till all the periods were *similar*, or terminated in the same column; this would have required *six more* places, or 12 places beyond the last figure of the non-recurring part, and these 12 places would have formed the *recurring part.* For in (i) the several recurring decimals consist of 1, 3, 3, 2, 4, 1 figures, and in (ii) of 3 and 4 figures; and the L. C. M. of these numbers is in each case 12; now if all the periods be considered to start together immediately after the last figure that does not recur, the *columns* will recur in the same relative order after every 12 places.

A **Recurring Decimal** may be *multiplied* by a **Finite Decimal**, giving a result sufficiently accurate for all practical purposes, by extending the period a few additional places, and then proceeding by the ordinary rules.

Example.—Multiply $.4\dot5 7\dot6$ by 23.4, giving the result true to 4 places.

$$\begin{array}{rr} .4576576 & \\ 23.4 & \\ \hline 18306304 & \\ 13729728 & \\ 9153152 & \\ \hline 10.7091\cancel{8784} & \end{array} \qquad\qquad \begin{array}{r} .4576576 \\ 23.4 \\ \hline 18306306 \\ 137297297 \\ 915315315 \\ \hline 10.70918918 \end{array}$$

The first result (10.7091) is true to 4 places; the second result $(10.7\dot0 9 1\dot8)$, obtained by extending the periods in each product, is strictly accurate.

A **Recurring Decimal** may be *divided* by a **Finite Decimal** with perfect accuracy by extending the periods in the dividend until the quotient recurs.

Example.—Divide $.\dot2 9\dot6$ by 24.

$$24\left\{\begin{array}{l} 4\underline{).296296296} \\ 6\underline{).074074074} \end{array}\right.$$
$$.\dot0 1234567\dot9$$

A **Recurring Decimal** may be *multiplied* or *divided* by another **Recurring Decimal** by expressing both in the form of vulgar fractions, and performing the required operation, and then expressing the result in a decimal form.

Example.—(i.) Multiply $1.\dot6\dot3$ by $.8\dot3$; (ii.) Divide $6.\dot6$ by $3.\dot6\dot3$.

(i.) $1.\dot6\dot3 = \frac{63}{99} = 1\frac{7}{11} = \frac{18}{11}$.

$.8\dot3 = \frac{83-8}{90} = \frac{75}{90} = \frac{5}{6}$.

$$\therefore\quad 1.\dot6\dot3 \times .8\dot3 = \frac{\overset{3}{\cancel{18}}}{11} \times \frac{5}{\cancel{6}} = \frac{15}{11} = 1.\dot3\dot6.$$

(ii.) $6.\dot6 = 6\frac{6}{9} = 6\frac{2}{3} = \frac{20}{3}$.

$3.\dot6\dot3 = 3\frac{63}{99} = 3\frac{7}{11} = \frac{40}{11}$.

$$\therefore\quad 6.\dot6 \div 3.\dot6\dot3 = \frac{20}{3} \div \frac{40}{11} = \frac{\cancel{20}}{3} \times \frac{11}{\underset{2}{\cancel{40}}} = \frac{11}{6} = 1.8\dot3.$$

Ex. 63.

.) $.78\dot{5} + 13.\dot{5}62\dot{7} + \dot{0}\dot{9} + 25.4\dot{9}6\dot{3}$; $28856\dot{4} - 11.08\dot{7}$.

.) $20.070\dot{7} + 84.986\dot{5}4\dot{3} + .0005$; $33.5\dot{9}8\dot{7} - 32.\dot{6}$.

.) $.43\dot{2} + .\dot{4}3\dot{2} + .4\dot{3}\dot{2} + .432$; $4.\dot{3}\dot{2} - 2.57\dot{2}1\dot{5}$.

.) $.96\dot{5}\dot{4} \times .\dot{5}$; $.\dot{7}6923\dot{0} \times .\dot{5}\dot{2}$; $61.\dot{1}6\dot{5} \times .03\dot{6}$.

5.) $.3 \div .\dot{0}9$; $.\dot{0}9 \div .\dot{0}\dot{4}$; $.042 \div .0\dot{3}\dot{6}$; $234.\dot{6} \div .\dot{7}$.

6.) $.\dot{0}\dot{8} \div 1.\dot{5}3846\dot{1}$; $.\dot{9}0476\dot{1} \div 1.\dot{7}\dot{2}$; $.\dot{0}036\dot{9} \div \dot{0}027\dot{1}$.

7.) $\dfrac{2.8 \times 2.\dot{2}\dot{7}}{1.\dot{3}\dot{6}} + \dfrac{8.\dot{8} - 5.\dot{6}}{3.\dot{3} + 5.\dot{2}5\dot{9}}$; $\dfrac{15.0\dot{6}}{1.5} + \dfrac{9}{.\dot{4}\dot{5}} - \dfrac{8.\dot{9}6\dot{2}}{.\dot{3}}$.

8.) $\dfrac{.01\dot{8} + 01\dot{2} - 01\dot{3}}{.02\dot{8} + .5 + .01\dot{5}} \div \dfrac{.\dot{6} \times .7854}{1.8 \times .6545}$.

XXV.—REDUCTION OF DECIMALS.

To find the value of any Decimal Fraction of a given quantity.

Rule.—**Multiply** the Decimal by the number of the next 'er denomination which is equal to one of the given denination, pointing off as many decimal places as there in the multiplicand, (according to the rule for Multipliion of Decimals).

[f the given quantity be expressed in more than one denination, it should first be reduced to the lowest denomiion mentioned.

[f the given decimal be a **Recurring Decimal** it will, in st cases, be advisable to express it in the form of a lgar Fraction, and then find the value as already shown ge 96).

xample.—Find the value of (i.) £4.975 ; (ii.) 3.105 of an acre;) *.11 of 1 cwt. 8 qrs.* 4 lbs. ; (iv.) $.8\dot{3}$ of 5 days 6 hours.

$$\text{(i.)}\quad \begin{array}{r} \pounds \\ 4.975 \\ 20 \\ \hline 19.5\cancel{0}\cancel{0}\text{ s.} \\ 12 \\ \hline 6.0\text{ d.} \end{array} \qquad\qquad \text{(ii.)}\quad \begin{array}{r} \text{acre.} \\ 3.105 \\ 4 \\ \hline .42\cancel{0}\text{ ro.} \\ 40 \\ \hline 16.80\text{ po.} \end{array}$$

Ans. £4.975 = £4 19s. 6d.

$$\text{Ans. } \overset{\text{a.}}{3.105} = \overset{\text{a.}}{3}\ \overset{\text{r.}}{0}\ \overset{\text{p.}}{16.8}$$

(iii.) 1 cwt. 3 qrs. 4 lbs. = 200 lbs.

200 × .11 = 22.00 = 22 lbs.

$$\text{(iv.)}\ .8\dot{3} = \frac{83-8}{90} = \frac{75}{90} = \frac{5}{6}.$$

$$\text{Then } \frac{5}{6} \text{ of 5 days 6 hrs.} = \frac{5}{\cancel{6}} \times \frac{\overset{21}{\cancel{126}}}{1} = \overset{\text{hrs.}}{105} = \overset{\text{d.}}{4}\ \overset{\text{m.}}{9}$$

Ex. 64.

Find the value of

(1.) £.2875 ; £.634375 ; £.71875 ; £.78125.

(2.) .035 of £5 ; .00875 of £5 13s. 4d. ; $.31\dot{6}$ of £4 10s.

(3.) .025 of 25s. ; .875 of 17s. 6d. ; $2.3\dot{5}$ of 6s. ; $.\dot{8}\dot{1}$ of £9 8s. 10d.

(4.) .1875 cwt. ; .06125 gr. ; .175 ton ; .44 of 1 qr. 21 lbs. 8 oz.

(5.) .34375 yd. ; .0175 mile ; $11.\dot{7}\dot{2}$ of 7 miles 2 fur.

(6.) 11.15 acres ; 2.75 of 5 a. 1 r. 32 p. ; .4125 of 486 acres.

(7.) .38125 day ; .00625 day ; .325 of 7 hrs. 40 m. ; $.0791\dot{6}$ hrs. ; 365.24224 days.

(8.) 2.0945 cu. ft. ; .03125 gal. ; .075 of 3 bu. 2 pks.

(9.) Reduce 6 mètres to yds. ; 8 decimètres to feet ; 15 kilomètres to miles.

(10.) Express in avoirdupois weight, 5 kilogrammes ; 18 quintals ; 7125 grammes.

(11.) Reduce 116 hectares to acres ; 276 square mètres to sq. yds. ; 13.3 acres to roods.

(12.) Reduce 37 hectolitres to quarters ; 17.5 litres to gallons.

2. *To express one quantity as a decimal fraction of another.*

Rule.—Express the first quantity as a vulgar fraction of the second, (see page 99), and then reduce this fraction to *the decimal form.*

Example.—Express (i.) 14s. 3½d. as the decimal of a pound; (ii.) 4 lbs. 8 oz. to the decimal of 3 qrs. 12 lbs.

$$\text{(i.)}\ \frac{\overset{\text{s.}}{14}\ \overset{\text{d.}}{3\tfrac{1}{2}}}{20\ \ 0} = \frac{\overset{\text{halfpence.}}{343}}{480} = .714583.$$

$$\text{(ii.)}\ \frac{\overset{\text{lbs.}}{4}\ \ \overset{\text{oz.}}{8}}{96\ \ 0} = \frac{72}{1536} = \frac{9}{192} = \frac{3}{64} = .045625.$$

If the second quantity is of *one denomination*, we may sometimes conveniently proceed as follows:—

Divide the number of the lowest denomination in the first quantity by as many of that denomination as make one of the next greater; to this quotient prefix the given number, (if any), of the latter.

Divide this in like manner and so proceed, as in ordinary Reduction, till a decimal is obtained of the required name.

Example.—Reduce (i.) 14s. 3½d. to the decimal of a pound; (ii.) 4 lbs. 8 oz. to the decimal of 1 cwt.

$$\text{(i.)}\ \begin{array}{r|l} 4 & 2. \\ \hline 12 & 3.5 \\ \hline 20 & 14.291\dot{6} \\ \hline & .71458\dot{3} \end{array} \qquad\qquad \text{(ii.)}\ \begin{array}{r|l} 16 & 8. \\ \hline 28 & 4.5 \\ \hline 4 & .160\dot{7}1428\dot{5} \\ \hline & .04017\dot{8}5714\dot{2} \end{array}$$

Ex. 65.

(1.) Express 5s.; 2s. 6d.; 3s. 4d.; 7s. 6d.; 15s.; 16s. 8d.; 17s. 6d. as decimals of a pound.

(2.) 8s. 7½d.; 12s. 4½d.; 15s. 9d.; 18s. 6¾d.; 19s. 0¼d. as decimals of a pound.

(3.) 12s. 6¾d. : 15s. 7½d.; £1 8s. 9d.; £21 0s. 11¾d.; £5 9s. 0¼d.; £12 0s. 1½d. as decimals of a pound.

(4.) 3 qrs. 21 lbs.; 2 qrs. 13 lbs.; 1 qr. 16 lbs. 5 oz.; 7 cwt. 0 qrs. 4 lbs.; 3 cwt. 0 qrs. 16 lbs.; 3 qrs. 3 lbs. 1 oz. 12⅞ dr. as decimals of 1 cwt.

(5.) 1000 yds.; 7 fur.; 30 poles; 12232 feet; 3 fur. 10 po.; 5 ft. 6 in.; 27 miles 5 fur. 78½ yds. as decimals of a mile.

(6.) 23 hrs. 56 min. 4.09 sec., 9 hrs. 55 min. 50 sec. as decimals of a day: and 365 days 5 hrs. 48 min. 49.7; 365 days 6 hrs. 9 min. 9.6 sec. as decimals of a year.

(7.) 3 wks. 4 days 5 hrs. 6 min. as the decimal of a month: and 3 wks. 3 days 8¾ hrs. as the decimal of a year of 365¼ days.

(8). *5 lbs. 10 oz. 3 dwts.* 8 grs. troy as the decimal of a cwt. avoir

(9.) 3 fur. 10 po. 3 yds. 2 ft. as the dec. of 2½ miles.

(10.) 10s. 1½d. as the dec. of £10 13s. 4d.; 3s. 6d. as the dec. of £2 10s.; and 7½d. as the dec. of 7s.

(11.) 3 oz. 5 dwts. as the dec. of 1 lb. 3 oz.; 3 bu. 3½ pks. as the dec. of 5 quarters.

(12.) 3 roods 15 po. as the dec. of 3 a. 1 r. 10 p.; 2 ft. 3 in. as the dec. of 6 yds. 1 ft.; 3⅜ min. as the dec. of 3 hrs. 45 min.

Decimal Coinage.

According to the system of Decimal Coinage referred to on page 28, the Pound sterling is divided into *florins*, *cents*, and *mils*, which being respectively *tenths*, *hundredths*, and *thousandths* of a pound may always stand in the first, second, and third place of decimals.

Thus £12 6 fl. 8 c. 7 m. may be written £12.687.

Under such a system there will be no need of reduction, as the value can be read off at sight, and addition, subtraction, multiplication, and division of money would be the same as addition, subtraction, multiplication, and division of simple decimals.

Example.—(i.) Add together 7 fl. 5 c. 3 m.; £3 4 fl. 6 c. 9 m.; £7 9 fl. 2 c. 6 m.; £13 0 fl. 5 c.

£	fl.	c.	m.		£
	7	5	3	*or thus*	.753
3	4	6	9		3.469
7	9	2	6		7.926
13	0	5	0		13.050
25	1	9	8		25.198

(ii) Subtract £3 9 fl. 9 c. 6 m. from £8 7 m.

£	fl.	c.	m.		£
8	0	0	7	*or thus*	8.007
3	9	9	6		3.996
4	0	1	1		4.011

(iii.) Find the value of 37 pieces of cloth at £7 3 fl. 5c. the piece; also (iv.) the value of 1 piece, when 49 pieces cost £178 5 fl. 8 c.

(iii.)

$$\begin{array}{r} \pounds \\ 7.350 \\ \underline{37} \\ 51450 \\ \underline{22050} \\ 271.950 \end{array}$$

Ans. £271 9 fl. 5 c.

(iv.)

$$\begin{array}{r} \pounds \\ 49\left\{\begin{array}{l} 7\overline{)178.580} \\ 7\overline{)\ \ 25.511} \end{array}\right. \\ \overline{3.644} \end{array}$$

Ans. £3 6 fl. 4 c. 4 m.

Some of the advantages arising from a Decimal Coinage may be obtained even under the present system by expressing the given sums of money as decimals of a pound.

This may be done by the last rule (page 116); or true to three places of decimals, which will be sufficiently accurate for most practical purposes **at sight**, as follows:—

Write down the number of pounds to the left of the decimal point; then take half the number of shillings if *even*, or half the number of shillings less one if *odd*, and set it in the *first* place of decimals; turn the remaining sum into farthings, adding one for every sixpence or 24 farthings, and place the number so obtained in the second and third places.

Example.—Express £2 17s. 7¾d. as the decimal of a pound to three places, or as *pounds*, *florins*, *cents*, and *mils*.

at sight
£2 17s. 7¾d. = £2.882
= £2 8 fl. 8 c. 2 m.
accurately
$12\underline{)\ 7.75}$
$20\underline{)17.6458\dot{3}}$
$£2.88229\dot{6}$

Worked thus:—Set down the £2, then since 17s. is odd, set down the half of 16 that is 8 in the *first* place of decimals; turn 1s. 7¾d. into farthings = 79, add 3 for the three sixpences contained in them, and set down the resulting 82 in the *second* and *third* places; the answer is true within a third of a farthing.

The reason for halving the number of shillings is that any *even* number of shillings equals half that number of florins or *tenths* of a pound.

The reason for adding 1 *mil.* for every 6d. or 24 farthings, is that 6d. or 24 *farthings* = 25 *thousandths* of a pound.

This method is *absolutely accurate only* when the pence figure is 0 or 6.

The converse operation of finding **at sight** the value of the decimal of £1 to the nearest farthing is as follows:—

Double the figure in the first place of decimals for shillings; take the figures in the second and third places as farthings and reduce them to shillings and pence, first subtracting 1 for every 25.

If the second figure is 5 add 1s. and count the third figure as farthings.

This method is ***absolutely accurate only*** when the given decimal ends in 00, 25, 50, or 75.

Example.—Find the approximate value of £3.856.

at sight £3.856 = £3 17s. 1½d.

accurately £3.856
20
17.120
12
1.440
Ans. £3 17s. 1.44d.

Worked thus :—Set down the £3. Double the first figure 8 which will give 16s. Add 1s. as the second figure is 5, this will give 17s. Then the 6 taken as farthings will give 1½d. Therefore the value is £3 17s. 1½d ***nearly***.

The following examples will illustrate some of the applications of decimal arithmetic to our present money as well as to the **metric system** of weights and measures.

(i.) Multiply £57 16s. 6d. by 273.

£
£57 16s. 6d. = 57.825
273
173475
404775
115650
15786.225 = £15786 4s. 6d.

(ii.) Find the value of 11 cwt. 3 qrs. 21 lbs. at £11 13s. 7d. per cwt.

£
£11 13s. 7d. = 11.679
11
128.469

2 qrs.	$\frac{1}{2}$	5.8395
1 qr.	$\frac{1}{2}$	2.91975
14 lbs.	$\frac{1}{2}$	1.459875
7 lbs.	$\frac{1}{2}$	.7299375

139.4180625 = £139 8s. 4½d.

(iii.) Find the value of 52 hectolit. 25 lit. of wine at 2 fr. 45 c. per litre.

52.25
2.45
26125
20900
10450
128.0125 = 128 francs 1 centime.

Multiply the numbers together, point off as usual, the result will be the number of francs and decimals of a franc, of which only the first two places are required.

(iv.) If 115½ kilogrammes of sugar cost 159 fr. 39 c., what is the price of one kilogramme?

115.5)159.39(1.38 = 1 fr. 38 c.
115.5
4389
3465
9240
9240

Ex. 66.

Express with three decimal places or as *pounds, florins, cents*, and *mils*:—

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	1	13	7½	(2.)	4	0	3¾	(3.)	2	8	8½
(4.)	4	18	11¼	(5.)	3	14	5¼	(6.)	6	15	9½
(7.)	8	12	4¾	(8.)	17	19	1½	(9.)	20	9	2¼

Find the approximate value of

(10.) £3.625. (11.) £5.912. (12.) £7.945.
(13.) £15.780. (14.) £1.414. (15.) £17.186.
(16.) £36.485. (17.) £75.733. (18.) £19.674.

(19.) A person's income is 20824 fr. 50 c., how much may he spend daily so as to lay by 1250 francs at the year's end.

(20.) How often will a wheel 3½ mètres in circumference turn in going over a distance of 26 kilomètres, 369 mètres?

(21.) Find the railway fare from Lyons to Paris (507 kilomètres) at 13 c. per kilom.

(22.) Divide 372 fr. 4 c. among 131 persons.

Ex. 67.

Miscellaneous Examples in Decimals.

(1.) Divide ·03611 by 7.85; 2268 by ·006; and 21·6 by ·035.

(2.) Divide ·0567875 by 1·25, and multiply the result by 2·136.

(3.) Reduce $\frac{1001}{1000}$, $\frac{19}{128}$, $\frac{1}{1111}$, to decimals; and ·015625, $\cdot\dot{0}09\dot{9}$, to vulgar fractions.

(4.) Simplify (i.) (·004 ÷ ·025) × (124 ÷ ·0062).

(ii.) (8·315 × ·025) ÷ 3·14 + 2·015 — 4.1635).

(iii.) $$\frac{\cdot 02\dot{6} + \cdot 23\dot{8} - \cdot 45\dot{3}}{.07\dot{2} + \cdot 25\dot{6} - .37\dot{6}} - \left(\frac{\cdot 016}{.341} \times \frac{.0025}{.015}\right).$$

(5.) Divide the product of ·035 and ·0056 by ·00007. Also divide 5·04 by ·012, and 420 by ·420.

(6.) Reduce $\frac{711}{225}$, $\frac{3}{130}$, $\frac{\cdot 053}{640}$ to decimals; and ·016125, $\cdot 2\dot{1}4285\dot{7}$ to vulgar fractions.

(7.) Find the sum of 1·805, 130·55, ·00045 and 60. Find the product of ·000375 and ·0035. Subtract the latter result from the former. Divide the remainder thus obtained by 1·05.

(8.) Divide ·027 by 14·4; 1632 by ·0008; and 7 by ·091.

(9.) Reduce 7s. $8\frac{1942}{100000}$d. to the decimal of half a guinea; and express the difference between 2 778125 of 6s. 8d. and $\cdot 06\dot{3}$ of £13 2s. 6d. as the decimal of 2s. 3d.

(10.) Add together $\cdot\dot{1}4285\dot{7}$ of 7 guineas, and $2\cdot 01\dot{3}$ of 6s. 3d.

(11.) Find the value of $\frac{7}{9}$ cwt. and ·625 cwt.; also of 1·23456 of 30s. 6d.

(12.) Find the value of (i.) 2·125 of 17s. $5\frac{3}{4}$d.; (ii.) $\cdot 30069\dot{4}$ of a day.

(13.) Reduce 5 cwt. 1 qr. 15 lbs. 5 oz. to the decimal of a ton.

(14.) Multiply 76·4 by 35·01, and divide 77·14 by ·0032. Prove each by vulgar fractions.

(15.) Find the value of 1·43625 of £2 10s., and reduce £2 11s. $6\frac{3}{4}$d. to the decimal of £3.

(16.) Add together $\frac{3}{5}$ of 17s. 6d., $\frac{5}{27}$ of 21s., and $\frac{7}{32}$ of 10s., and reduce the result to the decimal of £4 19s. $5\frac{1}{2}$d.

(17.) Express $\frac{1}{5}$ of a florin + $\frac{3}{20}$ of 2s. 6d. as a decimal of £5; and find the value of 2.553125 of £10.

(18.) Simplify .0057 of £1 + 9.245 of 5s. 6d. — 1.26 of 2s. 3d.

(19.) Reduce ·0625 and $\cdot 00\dot{7}\dot{2}$ to vulgar fractions in their lowest terms.

(20.) Find the value of £·7625 + $\frac{5}{2}$ of 7s. 6d. — $\dot{5}\dot{4}$ of 8s. 3d. + $1.91\dot{6}$ of 8s. — $\frac{7\frac{1}{3}}{8\frac{1}{7}}$ of $\frac{5\frac{1}{7}}{7\frac{1}{3}}$ of 27s.

(21.) Simplify:—

$$\text{(i.)}\ \frac{.01728 \times .06\dot{3}}{.00\dot{4}2857\dot{1}}$$

$$\text{(ii.)}\ \frac{\cdot 0042 \times \cdot 144}{\cdot 0441 \times 00\dot{1}4285\dot{7}} + \frac{.\dot{5}04}{.432} \div \frac{\cdot 1728}{\cdot 00\dot{4}2857\dot{1} \times \cdot 063}.$$

$$\text{(iii.)}\ \left(\frac{7\frac{1}{2} + 3\frac{2}{3}}{75 \times 36.6} + \frac{2.\dot{3} \times 15}{2\frac{1}{3} \times 3\frac{3}{4}} + \frac{7.25}{11\frac{3}{5}}\right) \div 56\frac{1}{4}.$$

$$\text{(iv.)}\ \frac{\cdot 01\dot{8} - \cdot 01\dot{3} + \cdot 01\dot{2}}{.01\dot{5} + \cdot\dot{5} + \cdot 02\dot{8}} \times \frac{1.8}{\cdot\dot{6}} \times \frac{\cdot 7854}{\cdot 6545}.$$

(22.) Reduce $\frac{11}{125}$ to a decimal, and 2·084 to a vulgar fraction. Di- ide 3 by ·06, and 2·5 by ·64.

(23.) Express 12s. 6¾d.; 15s. 9¾d., and £4 13s. 4½d. as decimals of '1; and reduce 12 hrs. 55 min. 24 sec. to the decimal of a day.

(24.) Divide ·0$\dot{6}$7$\dot{2}$ by 2·13$\dot{4}$. What decimal fraction of 2s. 11⅛d. is s. 1⅓d.?

(25.) Reduce 3 qrs. 16 lbs 5⅓ oz. to the fraction and decimal of a wt., and £1 6s. 3d. to the fraction and decimal of £2 5s. 11¼d.

(26.) Reduce 3⅔s. to the decimal of $\frac{5}{21}$ of a guinea: and find the alues of ·232 of a cwt., and 4·0$\dot{1}$7$\dot{1}$ of a mile.

(27.) Find the difference between ·70323 of a pound, and 3·5646 of shilling; and reduce 14 hrs. 15 min. to the decimal of 3½ days.

(28.) Reduce 2 qrs. 11 lbs. 3⅕ oz. to the decimal of a cwt.

(29.) Find the value of ·75 of 6s. 8d. + 1·84375 of 4s. + 3.978$\dot{3}$ f 2s.; also of 1·$\dot{8}\dot{1}$ of 5⅓ days + 5·75 of a week — ¾ of 5⅝ hours.

(30.) How many lbs. Troy in 5 kilogrammes?

(31.) The sun's diameter is 111·454 times the equatorial diameter of he earth, which is 7925·648 miles; what is the diameter of the sun?

(32.) In a parish of 450 people, ·44 of the whole were men and oys, ·38 were women and girls, and the rest were children. How ıany were there of each?

(33.) The true length of a year is 365·242218 days. Express this n days, hours, minutes, and seconds.

(34.) Find the value of 603⅔ yards of cloth at 9s. 7⅝d. per yard, and ive the result in decimals of £1.

(35.) Find a half-year's rent of 22.791$\dot{6}$ acres of land, at £3.720 per nnum per acre.

(36.) Reduce one inch to the decimal of a mètre; and one kilo- ramme to the decimal of a ton.

XXVI.—PROPORTION.

Ratio is the *relation* which one quantity or number ears to another with respect to magnitude.

A **ratio** can only exist between quantities *of the same ind.*

We cannot compare, for instance, a week and a cwt.; a pound and an inch; a pint and a day; and say that one is either larger or smaller than the other.

The **ratio** of two quantities or numbers is found by *dividing one by the other*, and may be expressed, either as a fraction or by placing two dots (:) between the numbers.

Thus, the ratio of 36 *to* 72 may be written $\frac{36}{72}$, or 36 : 72, and is the same as the ratio of 1 to 2; that is, the second number is twice the first.

The *first* term of a ratio, that is the *dividend*, is called the **Antecedent.**

The *second* term, that is the *divisor*, is called the **Consequent.**

A ratio *always* represents an *abstract number*, though its terms may be concrete numbers, such as, pounds, cwt., yards, &c.; for the division of one concrete number by another *always* gives an abstract number, expressing the number of times one is contained in the other.

Proportion is the equality of ratios, that is, when two ratios are equal, the numbers composing them are said to be in proportion.

In other words, "Four quantities or numbers are said to be in proportion when the ratio of the first to the second, is the same as the ratio of the third to the fourth."

Thus, the ratios of 8 to 16, and 24 to 48 being the same (for $\frac{8}{16} = \frac{1}{2}$, and $\frac{24}{48} = \frac{1}{2}$), the numbers 8, 16, 24, 48, are in proportion.

A proportion is expressed by placing four dots (: :) between the two ratios of which it is composed; thus, 8 : 16 : : 24 : 48 which is to be read 8 *is to* 16, *as* 24 *to* 48; or *as* 8 *is to* 16, *so is* 24 *to* 48.

A proportion may always be converted into a pair of equal fractions and a pair of equal fractions into a proportion.

When four numbers are in proportion, **the product of the Extremes** (*i.e.* the first and fourth) **is equal to the product of the Means** (*i.e.* the second and third).

Hence every proportion can be turned into an equation.

Thus, if 4 : 9 : : 8 : 18;

Then $4 \times 18 = 8 \times 9 = 72$.

For since 4 : 9 is the same ratio as 8 : 18 ; $\therefore \frac{4}{9} = \frac{8}{18}$.
Multiplying both fractions by 9 × 18 we have

$$\frac{4 \times \cancel{9} \times 18}{\cancel{9}} \quad . \quad \frac{8 \times 9 \times \cancel{18}}{\cancel{18}} \quad \therefore 4 \times 18 = 8 \times 9.$$

If any three terms of a proportion are known, the fourth can always be found, from the above equation, as follows :—

To find one of the extremes.—Divide the product of the two means by the given extreme.

To find one of the means.—Divide the product of the two extremes by the given mean.

Example.—Let x represent the unknown term in the following proportions.

(i.) $x : 8 :: 10 : 16$; $x = \frac{8 \times 10}{16} = \frac{80}{16} = 5.$

(ii.) $5 : x :: 10 : 16$; $x = \frac{5 \times 16}{10} = \frac{80}{10} = 8.$

(iii.) $5 : 8 :: x : 16$; $x = \frac{5 \times 16}{8} = \frac{80}{8} = 10.$

(iv.) $5 : 8 :: 10 : x$; $x = \frac{8 \times 10}{5} = \frac{80}{5} = 16.$

Factors common to an extreme and mean term may be cancelled thus :—

$$\text{(i.) } x = \frac{\overset{5}{\times \cancel{10}}}{\underset{2}{\cancel{16}}} = 5 \text{ ; (iii.) } x = \frac{5 \times \overset{2}{\cancel{16}}}{8} = 10$$

Ex. 68.

Find a first proportional to :—

(1.) 21, 18, 54; 8, 35, 20; 14, 12, 28.

(2.) $\frac{5}{6}, \frac{7}{22}, \frac{35}{66}$; $7\frac{2}{3}, \frac{11}{30}, 7\frac{50}{100}$; $\frac{3}{4}, \frac{5}{8}, \frac{5}{16}$.

(3.) 4.2, 6.6, 7.7 ; 2.89, .171, .0323 ; 8.8, 1.76, 5.612.

Find a second proportional to :—

(4.) 8, 5, 15 ; 7, 21, 54 ; 9, 7, 136.

(5.) $3\frac{1}{4}, 8\frac{1}{21}$, 13 ; $2\frac{1}{7}$, 3, 4 ; $2\frac{3}{4}, 8\frac{4}{5}, 28\frac{4}{25}$.

(6.) 3 75, 5, 2 ; 2.625, 3.5, 4 ; 1.11, 3.7, .875.

Find a third proportional to :—

(7.) 6, 14, 28 ; 16, 21, 105 ; 15, 13, 117.

(8.) $3\frac{2}{9}$, $6\frac{3}{7}$, $38\frac{4}{7}$; $\frac{1}{2}$, $\frac{5}{11}$, $2\frac{6}{7}$; $7\frac{7}{8}$, $6\frac{9}{10}$, $21\frac{1}{3}$.

(9.) .27, 11.7, 91 ; 1.001, .5005, 1.8 ; $.0\underset{\cdot}{0}3$, 7, .11.

Find a fourth proportional to :—

(10.) 49, 56, 28 ; 36, 174, 144 ; 1008, 819, 112.

(11.) $\frac{8}{9}$, $\frac{7}{15}$, $\frac{14}{17}$; $6\frac{1}{10}$, $6\frac{3}{5}$, $9\frac{4}{7}$: $4\frac{1}{12}$, $14\frac{7}{24}$, $3\frac{73}{168}$.

(12.) 10.8, 10, 4.5 ; 5.06, .0017, .033 ; 19.7, 34.5, 1.01

XXVII.—SIMPLE PROPORTION.

SIMPLE PROPORTION, or, as it is often termed, the RULE OF THREE, is merely the method of finding the fourth term of a proportion, the first three of which are known.

It is convenient, and therefore usual, to make the unknown term the fourth term, but it might stand in either place provided the other terms were arranged accordingly.

In every Proportion-sum three numbers or quantities are given, two of which are of the same kind, and what is required is, to find a fourth number or quantity of the same kind as the third, and just as many times larger or smaller than the third, as the second is larger or smaller than the first.

As a ratio can only exist between quantities of the same kind (page 123), it is evident that the first and second terms of the proportion must be the two given quantities of the same kind reduced to the same denomination; and that the fourth term will always be of the same denomination as the third.

The proper arrangement of the three given terms is called *stating* the sum, and the rule for doing so is deduced from the foregoing observations.

Rule.—1st. Place in the third term the quantity or number which is of the same kind as the answer required.

2nd. Consider whether the answer should be greater or less than the third term; if greater, put the greater of the other two quantities in the second place; if less, the less.

3rd. Set the remaining quantity in the first place.

4th. Reduce the first and second terms *to the same name*, and the third, if necessary, to the lowest denomination mentioned in it.

5th. Multiply the second and third terms together and divide the product by the first; the quotient will be the answer in *the same name that the third term was reduced to.*

The *working* may be arranged in various ways, as seen in the following examples.

Example I.—If 2 tons of coals cost 44s. what will 17 tons cost?

$$\begin{array}{ccccccc}
\text{tons.} & & \text{tons.} & & \text{s.} & & \text{s.} \\
2 & : & 17 & :: & 44 & : & x
\end{array}$$

$$x = \frac{\overset{22}{\cancel{44}} \times 17}{\cancel{2}} = \overset{\text{s.}}{374} = \overset{\pounds\ \ \text{s.}\ \ \text{d.}}{18\ \ 14\ \ 0}$$

Here, price being wanted, we place the given price (44s.) in the third term; then, as 17 tons will cost more than 2 tons, we put 17, which is the greater number in the second term and 2 in the first; the fourth term x represents the *unknown* price. We then proceed as in the fourth example, page 128.

Example II.—If 2 cwt. 3 qrs. 21 lbs. of sugar cost £12 3s. 4d. what will be the cost of 17 cwt. 2 qrs. 14 lbs?

$$\begin{array}{rrrcrrrcrrrcr}
\text{cwt.} & \text{qrs.} & \text{lbs.} & & \text{cwt.} & \text{qrs.} & \text{lbs.} & & \pounds & \text{s.} & \text{d.} & & \pounds \\
2 & 3 & 21 & : & 17 & 2 & 14 & :: & 12 & 3 & 4 & : & x \\
& 4 & & & & 4 & & & & & 6 & & \\
\hline
& \multicolumn{2}{l}{11 \text{ qrs.}} & & & \multicolumn{2}{l}{70 \text{ qrs.}} & & 73 & 0 & 0 & & \\
& 28 & & & & 28 & & & & & & & \\
\hline
& \multicolumn{2}{l}{\cancel{329} \text{ lbs.}} & & & \multicolumn{2}{l}{\cancel{1974} \text{ lbs.}} & & & & & & \\
& & & & & 6 & & & & & & &
\end{array}$$

Here £12 3s. 4d. or £12⅙ is set in the third term, since price was wanted; and as the 17 cwt., &c., would cost more than 2 cwt., &c., it is *put in the second term.*

Then the first and second terms are reduced to lbs.; and as the first is contained in the second, 6 times, we may cancel 329 and 1974, setting 6 below the latter. Lastly, £12 3s. 4d. × 6 = £73.

Or, the work might have been arranged thus :—

$$x = \frac{\overset{\text{cwt.}}{17}\quad \overset{\text{qrs.}}{2}\quad \overset{\text{lbs.}}{14}}{2\quad\ \ 3\quad\ 21} \times 12\tfrac{1}{6} = \frac{\overset{\cancel{6}}{\cancel{1974} \times 73}}{\cancel{329} \times \cancel{6}} = £73.$$

Example III.—A bankrupt owes £535 10s. 5d., and his assets are £321 6s. 3d., what will a creditor receive to whom he owes £25 8s. 9d.?

$$\begin{array}{lccccccc}
\text{Here} & \text{debt} & : & \text{debt} & :: & \text{payment} & : & \text{payment.} \\
 & \overset{\text{£}}{535}\ \overset{\text{s.}}{10}\ \overset{\text{d.}}{5} & : & \overset{\text{£}}{25}\ \overset{\text{s.}}{8}\ \overset{\text{d.}}{9} & :: & \overset{\text{£}}{321}\ \overset{\text{s.}}{6}\ \overset{\text{d.}}{3} & : & \overset{\text{£}}{x} \\
 & \underline{20} & & \underline{20} & & \underline{20} & & \\
 & 10710\text{s.} & & 508\text{s.} & & 6426\text{s.} & & \\
 & \underline{12} & & \underline{12} & & \underline{12} & & \\
 & \cancel{128525}\text{d.} & & \cancel{6105}\text{d.} & & \cancel{77115}\text{d.} & & \\
 & \cancel{25705} & & 1221 & & \cancel{15423} & & \\
 & \cancel{5141} & & \underline{3} & & 3 & & \\
 & & & 12\overline{)3663} & & & & \\
 & & & 2{,}0\overline{)\ 30{,}5\ \ 3} & & & & \\
 & & & \overline{£15\ \ 5\text{s.}\ \ 3\text{d.}} & & & &
\end{array}$$

In this example the first term has been cancelled against the second and third.

Example IV.—If a man can walk $9\frac{3}{8}$ miles in $2\frac{3}{4}$ hours, how far can he walk in $6\frac{1}{2}$ hours?

$$\begin{array}{lccccccc}
\text{Here} & \text{time} & : & \text{time} & :: & \text{distance} & : & \text{distance.} \\
 & \overset{\text{hrs.}}{2\frac{3}{4}} & : & \overset{\text{hrs.}}{6\frac{1}{2}} & & \overset{\text{miles.}}{9\frac{3}{8}} & : & \overset{\text{miles.}}{x}
\end{array}$$

$$x = 9\tfrac{3}{8} \times 6\tfrac{1}{2} \div 2\tfrac{3}{4} = \frac{75}{\underset{2}{\cancel{8}}} \times \frac{13}{2} \times \frac{\cancel{4}}{11} = \frac{975}{44} = 22\tfrac{7}{44} \text{ miles.}$$

It will be seen from the above examples and from many other in the following exercises, that questions in Simple Proportion, or Rule of Three, are not confined to questions of *cost* or *quantity*, but include all those in which the idea *of proportion* is involved. Thus, for instance, a man's *work* *wages*, will be in proportion to the *time* he is employed;—the *rent* of a house to the *time* it is occupied;—the *weight* of a loaf to the *price* of flour;—a *tax* to the *rental* or *income;*—*distance* travelled to the *uniform rate* of travelling, &c., &c.

Many questions in which the first term of the proportion is 1 may be readily solved by Compound Multiplication or Practice (see Exercises 27, 30, 35, 36), which will furnish, if needful, additional exercises in Proportion.

Ex. 69.

(1.) If 19 cwt. of sugar cost £57, how many cwt. can be bought for £111?

(2.) If 27 lbs. of coffee cost £1 12s. 3d., how much will 76 lbs. cost?

(3.) If 4 cwt. 0 qrs. 2 lbs. of tea cost £105 18s. 9½d., what is the price of 1 lb.?

(4.) If 15 men can mow a field in 30 hours 15 minutes, how long will it take 9 men to mow it?

(5.) If the rent of 9 acres of land is £5 12s., what is the rent of 72 acres 2 roods 10 poles at the same rate?

(6.) If 3 cwt. 24 lbs. of sugar cost 10 guineas, how much can be bought for £6 13s.?

(7.) If 1 cwt. 2 qrs. 15 lbs. cost £6 13s. 4d., what will 4 cwt. 3 qrs. 23 lbs. cost?

(8.) A man working 10 hours a day completes a piece of work in 12 days, in how many days could he do it, working 8 hours a day?

(9.) What quantity of corn can be bought for £42, at the rate of 6s. per bushel?

(10.) What is the yearly rent of 547 acres of land, at 15s. 6d. the acre?

(11.) What is the amount of tax upon £763 15s., at the rate of 3s. 6d. per pound sterling?

(12.) At £1 7s. 8d. the acre, what is the rent of 173 acres 2 roods 14 poles of land?

(13.) If 7 cwt. 1 qr. of sugar cost £26 10s. 6d., what is the price of 43 cwt. 2 qrs.?

(14.) What is the value of a bar of pure silver weighing 73 lbs. 5 oz. 15 dwt., at the rate of 5s. 9d. per ounce?

(15.) The debts of a bankrupt amount to £977, and his property to £420 6s. 3¼d., how much per pound can he pay?

(16.) What cost 43 qrs. 5 bushels of corn, at £1 8s. 6d. the quarter?

(17.) If 1000 sovereigns weigh 21 lbs. 5¾ ozs. (troy), what will 768 sovereigns weigh?

(18.) If the carriage of 3 cwt. 50 lbs. for 135 miles cost 15s., how far ought 5 cwt. 19 lbs. to be carried for the same money?

(19.) How much sugar may be bought for £187 4s., when 3 cwt. 1 qr. cost £13 17s. 4d.?

(20.) Find the expense of lodgings for 3 mo. 2 wks. 5 days, at a guinea per week?

(21.) How much must be given for 85 quarters of wheat, when 5 qrs. 3 bus. cost £10 8s. 8¾d.?

(22.) What will be the charge of keeping 20 horses for a year, at the rate of 14½d. per day for each horse?

(23.) If I spend 20 guineas in 14 days, what must my income be to enable me to lay by £200 per annum?

(24.) If 5 yards of cloth cost 14s. 2d., what must be given for 9 pieces, each measuring 21 yds. 1 qr.?

(25.) If an ounce of gold cost 4 guineas, what is the value of a grain?

(26.) How many yards, worth 4s. 2¼d. a yard, must be given in exchange for 402 yards at 3s. 5½d. a yard?

(27.) If a twopenny loaf weigh 1 lb. 12 oz. when wheat sells at 50s. per quarter, what should it weigh when wheat sells at 70s.?

(28.) Bought a silver tankard, weighing 1 lb. 7 oz. 14 dwts., what did it cost me at 6s. 4d. the ounce?

(29.) How many steps of 2 ft. 9 in. each must be taken in a minute, in order to walk at the rate of 4 miles an hour?

(30.) What will 20 pieces of lead cost, each weighing 1 cwt. 12 lb., at the rate of 16s. 4d. per cwt.?

(31.) What length of paper ¾ of a yard wide will be required to cover a wall 15 ft. 8 in. long by 11 ft. 3 in. high?

(32.) If I exchange 126 gals. of sherry, worth 4s. 4d. per gal. for old port, worth 6s. 6d. per gal. how much port am I to receive?

(33.) How much tea, at £13 17s. per cwt. is equal in value to 8 cwt. 2 qrs. 12 lbs. of coffee at £3 9s. 3d. per cwt.?

(34.) If 756 yards of net cost £14, how many Flemish ells can I purchase for 6s. 8d.?

(35.) How many yards of cloth can I buy for £20 6s. 8d. if 3 English ells cost £3 3s. 4d.?

(36.) If I pay 3s. 9d. per hundred for quills, how many can I buy for £103 2s. 6d.?

(37.) If 9 English ells of cloth cost £13 4s., how many French ells can I buy for £123 4s.?

(38.) If 7 yards 2 qrs. of damask cost £3 15s., how many English ells can I buy for £71 5s.?

(39.) If 18 a. 1 r. 20 p. of land are let for £84, what would be the rent of 10 a. 3 r. 30 p. of the same land?

(40.) If a box of soap containing 2 cwt. 2 qrs. 7 lbs. costs £5 7s. 10½d. what will be the cost of 12 boxes each 2 cwt. 1 qr. 18 lbs.?

(41.) If a staff 5 feet high casts a shadow 7 ft. 2 in. long, what is the height of a steeple which casts a shadow 215 feet long?

(42.) A wedge of gold weighing 14 lbs. 3 oz. 8 dwt., is valued at £514 4s., what is the value of an ounce?

(43.) If, by working 7½ hours a day, a man can do a piece of work in 14 days, how many hours a day must he work in order to do it in 8¾ days?

(44.) A draper purchased 28 pieces of cloth for £1569 17s. 4d., at 9s. 8d. per yard, how many yards were in each piece?

(45.) A bankrupt's debts are £1051 17s. 6d. and his assets are £636 10s. 6d., what will a creditor *receive* in composition of a debt of £120?

(46.) If 77 men can execute an order in 30 days, how many more men must be engaged to complete it in 22 days?

(47.) What will a creditor *lose* on a debt of £4625, if he receives three dividends of 3s. 2¼d., 2s. 4½d., and 4s. 2¼d. in the £?

(48.) If a garrison of 1500 men, having provisions for 5 weeks, be reinforced with 500 men, how long will the provisions last?

(49.) A person paid as income tax £108 1s. 4½d. when the tax was 9d. in the pound, what was his income?

(50.) If 25 yards of railway iron weigh 14 cwt. 2 qrs. 1 lb., what weight of iron will be needed for a railway 36 miles in length?

Ex. 70.

(1.) If the carriage of $21\frac{7}{16}$ cwt. for 40 miles come to £1 3s. 6d., how far ought $24\frac{1}{2}$ cwt. to be carried for the same money?

(2.) If $\frac{5}{14}$ of an estate be worth £1282 3s. 4d., what is the value of $\frac{3}{8}$ of it?

(3.) What is the value of $\frac{1}{11}$ of $\frac{1}{12}$ of a vessel, if a person who owns $\frac{3}{11}$ of it sells $\frac{1}{9}$ of $\frac{7}{8}$ of his share for £350?

(4.) If $\frac{3}{7}$ of a cwt. of flour be worth $15\frac{3}{8}$s. what is the value of $\frac{1}{3}$ of a cwt.?

(5.) If $18\frac{2}{9}$ oz. cost 35 $\frac{7}{8}$s. what will 2 qrs. $12\frac{1}{3}$ lbs. cost?

(6.) If $\frac{3}{4}$ of a pound cost $2\frac{3}{5}$s., what will be the price of $11\frac{7}{12}$ cwt.?

(7.) If $\frac{5}{7}$ of a cwt. cost $\frac{6}{19}$ of £50, what will $\frac{2}{3}$ of 1 qr. cost?

(8.) If $\frac{2}{3}$ of an English ell cost 9s. $8\frac{2}{3}$d., how many yards can be purchased for £$\frac{7}{12}$?

(9.) If a person can walk $\frac{1}{4}$ of $1\frac{10}{11}$ of a mile in $\frac{1}{5}$ of an hour, how far can he walk in $\frac{1}{15}$ of $2\frac{3}{4}$ hours?

(10.) If 45 men can do a piece of work in $35\frac{7}{20}$ days, how many more will be required to do it in $29\frac{1}{5}$ days?

(11.) If $\frac{6}{11}$ of a ton cost £90, what will be the cost of $\frac{13}{15}$ of a cwt.?

(12.) If $\frac{2}{3}$ of $\frac{5}{192}$ of 20 lbs. cost $1\frac{3}{50}$d., how many lbs. can be bought for 19s. $10\frac{1}{2}$d.

(13.) If 1.26 cwt. can be bought for £11.7 how many cwt. can be bought for £22.75?

(14.) If a railway carriage proceeds at the rate of 3.5 miles in 7.8 minutes, how long will it take to reach a distance of 212 miles 2 fur. 3 p.?

(15.) If 8 cwt. 3 qrs. $16.\dot{3}$ lbs., cost £7.625, what will 4.06 cwt. cost?

(16.) If $7\frac{3}{4}$ yards of cloth cost £2.6375, what will be the cost of $140\frac{1}{2}$ yards?

(17.) If 1 lb. 8 oz. of coffee cost £.175, what will 326.25 lbs. cost?

(18.) How many yards of lining .75 yds. wide, will be required to line 55.75 yards of stuff .875 yards wide?

(19.) If $3.\dot{3}\dot{6}$ lbs. cost £.1875, what is the value of 6.16 lbs.?

(20.) What is the cost of .075 of a ton of coffee, when .0625 lbs. cost $.058\dot{3}$s.?

(21.) How many kilogrammes of tea at 8 fr. 75 c. per kilog. must be given in exchange for 700 kilog. of sugar at 1 fr. 35 c. per kilog.?

(22.) If a person travels 3 kilomètres 45 mètres in .975 of an hour, *how long* will he take in going $29\frac{1}{2}$ kilomètres?

(23.) If $37\frac{1}{2}$ hectolitres cost 562 frs. 50 c., what will be the cost of $5\frac{1}{2}$ litres?

(24.) If 3 hectolitres 96 litres of seed are required for 3 ares of land, how many will be required for 2 hectares 2 ares?

XXVIII.—COMPOUND PROPORTION.

Compound Proportion, or Double Rule of Three is so-called because there are two more questions in Simple Proportion combined into one.

In **Simple Proportion** we have to find a number or quantity to which a given number or quantity shall have a **single** given ratio.

In **Compound Proportion** we have to find a number or quantity to which a given number or quantity shall have a ratio **compounded** *of two or more* given ratios.

A ratio is said to be *compounded* of two or more ratios, when it is *the product* of those ratios.

Thus the ratio of 56 : 63 may be said to be compounded of the three ratios 2 : 3; 4 : 5; 7 : 9;—for, if we express these ratios as fractions and multiply them together, we obtain $\frac{56}{135}$ as the product, which is the ratio of 56 : 135.

$$\tfrac{2}{3} \times \tfrac{4}{5} \times \tfrac{7}{9} = \tfrac{56}{135}.$$

Statements in Compound Proportion are written in the following form:—

$$\left.\begin{array}{ccc} 2 & : & 3 \\ 4 & : & 5 \\ 7 & : & 9 \end{array}\right\} :: 56 : 135.$$

Now $56 \times 3 \times 5 \times 9 = 135 \times 2 \times 4 \times 7.$

Or, **The product of the means = the product of the extremes**, as in Simple Proportion.

In questions involving Compound Proportion one of the given numbers or quantities is of the same kind as the number or quantity required, and the others, taken two and two, are like one another.

In stating the question the same method may be followed as in Simple Proportion, taking the single given number as the third term in successive proportions of which *the other pairs* of numbers are, respectively, the first and second terms. Hence the following rule:—

RULE.—1st. Put in the third term the quantity or number which is of the same kind as the answer required.

2nd. Then take *each pair* of numbers of the same kind, *separately*, and consider, in reference to them alone, whether the answer should be greater or less than the third term; if greater put the greater of each pair in the second place; if less, the less; and set the remaining one in the first place.

3rd. Reduce each pair of first and second terms *to the same name*, and the third, if necessary, to the lowest denomination mentioned in it.

4th. Multiply all the second terms and the third term together, and divide their product by the product of all the first terms; the quotient will be the answer *in the same name that the third term was reduced to.*

EXAMPLE I.—If 20 horses eat 60 bushels of corn in 12 days, how many bushels will 24 horses require for 8 days?

$$\left.\begin{array}{ccc} \text{horses} & & \text{horses} \\ 20 & : & 24 \\ \text{days} & & \text{days} \\ 12 & : & 8 \end{array}\right\} :: \begin{array}{ccc} \text{bush.} & & \text{bush.} \\ 60 & : & x \end{array}$$

$$x = \frac{\overset{3}{\cancel{60}} \times \overset{6}{\cancel{24}} \times 8}{\cancel{20} \times \underset{4}{\cancel{12}}} = 48 \text{ bushels.}$$

Here quantity being required, put the given quantity in the third term; then, as 24 horses will require *more* than 20 horses *in the same time*, place the *greater* number of horses in the 2nd term. Again, since a *less* quantity will be eaten by the *same number of horses* in 8 days *than in 12 days*, place the less number of days in the 2nd term.

XAMPLE II.—If 30 cwt. are carried 12 miles for £4 13s. 4d., how ught 56 cwt to be carried for £6 10s. 8d.?

$$\left.\begin{array}{ccc}\text{cwt.} & & \text{cwt.}\\ 56 & : & 30\\ \pounds & & \pounds\\ 4\tfrac{2}{3} & ; & 6\tfrac{8}{15}\end{array}\right\}\begin{array}{ccc}\text{miles} & & \text{miles.}\\ ::\ 12 & : & x\end{array}$$

$$x=12\times30\times6\tfrac{8}{15}\div56\times4\tfrac{2}{3}.$$

$$=\frac{\overset{3}{\cancel{12}}\times\overset{\cancel{2}}{\cancel{30}}\times\overset{\cancel{7}}{\cancel{98}}}{\cancel{15}}\times\frac{3}{\underset{\underset{\cancel{4}}{\cancel{8}}}{\cancel{56}}\times\cancel{14}}=9\text{ miles.}$$

ere distance being required, we set the given distance, 12 miles in hird term; then as 56 cwt. should evidently be carried a less dis e than 30 cwt. *for the same money*, we put the less quantity in the term; again, since *the same quantity* should evidently be carried er for £6 10s. 8d. than for £4 13s. 4d. we put the greater sum in nd term.

any questions in Compound Proportion may easily be arranged e statement, and worked as a Simple Proportion; thus the pre g examples might have been stated as follows:—

$$\begin{array}{lccccccc} & \text{cause} & : & \text{cause} & :: & \text{effect} & : & \text{effect}\\ \text{(i.)} & 20\times12 & : & 24\times8 & :: & 60 & : & x\end{array}$$

$$x=\frac{\overset{2}{\cancel{24}}\times8\times\overset{3}{\cancel{60}}}{\cancel{20}\times\cancel{12}}=48\text{ bushels.}$$

$$\begin{array}{lccccccc} & \text{cause} & : & \text{cause} & :: & \text{effect} & : & \text{effect}\\ & & & & & \pounds\ \ \text{s.}\ \ \text{d.} & & \pounds\ \ \text{s.}\ \ \text{d.}\\ \text{(ii.)} & 30\times12 & : & 56\times x & :: & 4\ \ 13\ \ \ 4 & : & 6\ \ 10\ \ \ 8\end{array}$$

en (as in (iii.), page 125)

$$x=\frac{\overset{\pounds\ \ \ \text{s.}\ \ \ \text{d.}}{6\ \ 10\ \ \ 8}\times30\times12}{4\ \ 13\ \ \ 4\times56}=\frac{\overset{\cancel{14}}{\cancel{1568}}\times\cancel{30}\times\overset{3}{\cancel{12}}}{\cancel{1120}\times\underset{\cancel{4}}{\cancel{56}}}=9\text{ miles.}$$

use and effect are always proportional to each other.

(i.) 20 horses, eating for 12 days, are the *cause*, the consumption bushels the effect; what then will be the effect, of which 24 , eating for 8 days, are the cause?

(ii.) *the carriage of* 30 cwt. for 12 miles is the known cause, the effect of which is the expenditure of £4 13s. 4d.; the carriage of 56 cwt. for *a certain* distance must be the cause, the effect of which is the expenditure of £6 10s. 8d.

Ex. 71.

(1.) If 7 men reap 15 acres of wheat in 3 days, how many men will reap 25 acres in 5 days?

(2.) If I get 8 oz. of bread for 6d. when wheat is 15s. a bushel, what does a bushel of wheat cost, when I get 12 oz. of bread for 4d.

(3.) If 6 horses can plough 17½ acres in 4 days, how many acres will 54 horses plough in 2¼ days?

(4.) If 12 men in 7 days reap 63 acres of wheat, in how many will 15 men reap 112½ acres?

(5.) If 9 persons spend £39 6s. in 8 days, in how many days will 11 persons spend £72 1s. at the same rate?

(6.) If 3000 copies of a book of 11 sheets require 66 reams of paper, how much paper will be required for 5000 copies of a book of 12½ sheets?

(7.) If 10 men reap 20 acres of corn in 4 days of 8 hours each, in how many days of 12 hours each will 14 men reap 84 acres?

(8.) If 14 horses plough 36 acres in 9 days, how many acres will 42 horses plough in 7⅓ days?

(9.) If 42 horses in 20 days eat 10 quarters of oats, how many horses will consume 16 quarters in 24 days?

(10.) If the penny loaf weigh 9 oz. when wheat costs 6s. 3d. per bushel, what ought it to weigh when wheat costs 8s. 4d. per bushel?

(11.) If the carriage of 5 cwt. for 100 miles cost 6s. 4d., what will the carriage of 2½ tons cost for carrying 150 miles?

(12.) If 12 men, working 10 hours a-day, can reap 64 acres in 8 days, how much will 48 men, working 12 hours a-day, reap in 20 days?

(13.) If, when wheat is 5¾s. per bushel, the sixpenny loaf weigh 5¼ lbs., what will 52½ lbs. cost when wheat is 7s. 8d. per bushel?

(14.) If I have 2½ cwt. carried 84 miles for 25s., how far can I have 18⅔ cwt. carried for £5?

(15.) How many hours a-day must 42 boys work, to do in 45 days what 27 men can do in 28 days of 10 hours each, the work of a boy being half that of a man?

(16.) If it cost £59 2s. 1½d. to keep 3 horses for 7 months, what will it cost to keep 2 horses for 11 months?

(17.) If 50 labourers dig a trench 440 yards long, 3⅓ feet wide, and

2½ feet deep, in 32 days of 9 hours each, how many men can dig a trench a mile long, 2⅓ feet deep, and 3½ feet wide, in 72 days of 8 hours each?

(18.) If 12 men can dig a trench 40 yards long, and 4 feet wide, in 10 days of 8 hours each, how many hours a day must 55 men work, in order to dig a trench of the same depth, 220 yards long, and 5 feet wide, in 18 days?

(19.) If 27 men, working 10 hours a day, perform a piece of work in 14 days, how many boys will perform the same work in 15 days, working 7 hours a day, if the work of a boy equal half the work of a man?

(20.) If 150 men dig a trench 230 yards long, 4 wide, and 10 deep, in 12 days, working 10 hours a day, in how long a time will 50 men dig a trench half as long again, twice as wide, and of the same depth, working 12 hours a day.

(21.) If an iron bar, 3 feet long, 2½ inches broad, and ⅞ inch thick, weigh 22½ lbs., what will a bar 7 feet long, 1⅜ inches broad, and 2¼ inches thick, weigh?

(22.) If in 30⅚ days, of 6⅜ hours each, 9 men build a wall 100 yards long, and 12 feet high, how long will it take 12 men, working 12⅓ hours per day, to do 3⅓ times the work?

(23.) If 34 panes of glass, each 2½ feet long, by 15 inches wide, cost £3 3s. 9d., what will 40 panes cost, each 2 feet long, by 1 foot 6 inches wide?

(24.) If 7 iron bars, 3 feet long, $4{\cdot}\dot{3}$ inches thick, and 3 inches broad, weigh 147 lbs, what will be the weight of 6 bars, 2 feet long, 6 inches broad, and 3·25 inches thick?

(25.) If 6 men can reap 4·5 acres in 3·75 days of 9·5 hours each, in how many days will 9 men reap a field of 22·5 acres, working 12 hours a day?

(26.) If £75 gain £$3{\cdot}\dot{2}14285\dot{7}$ interest in 9 months, what will be the interest on £105 in one year?

XXIX.—PROPORTIONAL PARTS, OR DISTRIBUTIVE PROPORTION.

By **Distributive Proportion** is meant the division of a given number or quantity into **proportional parts,** that is, parts which have a given ratio to each other.

Questions involving a proportionate *distribution of property*, the *sharing of profits or losses* among partners, the *proportions of ingredients* in a compound, and the like, fall under this head. They may be solved by a proportion stated according to the following rule, which depends on the obvious principle, that "*the whole of any thing* is to *a part* as the *whole of any other thing* is to *a like part.*"

RULE.—*As* the **sum of the numbers** expressing the ratios *is to* **each separate number**, successively, *so is* the **whole sum**, or quantity to be divided, *to* **each separate part** of it.

There will thus be as many separate proportions as there are parts composing the whole.

N.B.—The ratios should be expressed in their lowest terms, which may be found by dividing each number by their G.C.M.

Ratios expressed by Vulgar Fractions should be expressed by equivalent ratios in whole numbers, which will be *the numerators* of the fractions, when reduced to a common denominator.

EXAMPLE I.—Divide £240 into 3 parts, having the same ratio as 3, 4, 5.

Here the sum of the numbers expressing the ratios is $3 + 4 + 5 = 12$; then

$$12 : 3 :: 240 : x; \qquad x = \frac{\overset{20}{\cancel{240}} \times 3}{\cancel{12}} = 60.$$

$$12 : 4 :: 240 : x; \qquad x = \frac{\overset{20}{\cancel{240}} \times 4}{\cancel{12}} = 80.$$

$$12 : 5 :: 240 : x; \qquad x = \frac{\overset{20}{\cancel{240}} \times 5}{\cancel{12}} = 100.$$

The three parts are £60, £80, £100.

It will frequently be found very convenient in practice, if, instead of multiplying the second and third terms together, and *then* dividing by the first, we begin, at once, by dividing the third term *(the given quantity)* by the first *(the sum of the numbers expressing the ratios)*, and *then* multiply the quotient so found by each of those numbers separately.

Example II.—A puts £940, B, £560, C, £840, into a business; they gain £351. What is each man's share of the gain?

The shares will be in the ratio of 940, 560, 840, or 47, 28, 42.

Then $47 + 28 + 42 = 117$; and $£\frac{351}{117} = £3$;

$$\left.\begin{array}{l} \therefore \text{ A's share} = 47 \times 3 = £141 \\ \text{B's share} = 28 \times 3 = £\ 84 \\ \text{C's share} = 42 \times 3 = £126 \end{array}\right\} = £351$$

Example III.—Divide 355 into parts which shall be in the ratio of $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}$.

Now $\frac{1}{3}, \frac{1}{5}, \frac{1}{7} = \frac{35}{105}, \frac{21}{105}, \frac{15}{105}$, and are in the ratio of 35, 21, 15.

Then $35 + 21 + 15 = 71$; and $\frac{355}{71} = 5$; $\therefore$ the three parts are:

$35 \times 5 = 175$; $21 \times 5 = 105$; $15 \times 5 = 75$.

Example IV.—Three farmers rent some grass land for £70. A puts in 50 cattle for 4 months, B, 40 for 7 months, C, 30 for 12 months. What part of the rent should each pay?

$$\begin{array}{r} 50 \times 4 = 200 \\ 40 \times 7 = 280 \\ 30 \times 12 = 360 \\ \hline 840 \end{array}$$

$£\frac{70}{840} = £\frac{1}{12}$.

Each man's rent must be in the *compound ratio* of the *number* of oxen and length of *time* they were on the land; the numbers expressing the ratios will therefore be 200, 280, 360.

	£	£	s.	d.
$\therefore$ A's share $= 200 \times \frac{1}{12}$	=	16	13	4
B's share $= 280 \times \frac{1}{12}$	=	23	6	8
C's share $= 360 \times \frac{1}{12}$	=	30	0	0
		£70	0	0

Ex. 72.

(1.) Divide 1200 into 4 parts, having the ratios of 11, 12, 13, 14; also into 4 other parts, having the ratios of $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$.

(2.) Three traders, A, B, C, put the following sums into a business: A, £500; B, £650; and C, £700: the year's profits are £555. Required each person's share of them.

(3.) A general imposed a contribution of £1400 on four villages, to be paid in proportion to the number of inhabitants contained in each; the first contained 500, the second 700, the third 800, and the fourth 1000 persons. What part had each village to pay?

(4.) Gunpowder is composed of nitre, charcoal, and sulphur; thus, nitre, 76 parts; charcoal, 14; sulphur, 10. How much of each is used in 1 cwt. of gunpowder?

(5.) Divide £2046 10s. 6d. among three railway companies, in the proportions of 27, 31, and 45.

(6.) Pewter is composed of 112 parts of tin, 15 of lead, and 6 of brass. How much of each is there in 1 ton of pewter?

(7.) Pure water contains 88·9 parts of the gas called oxygen, and 11·1 parts of the gas called hydrogen. What weight of each is there in a cubic foot of water, which weighs 1000 oz.?

(8.) A bankrupt owes to 3 creditors £312 10s., £418 6s. 8d., £596 3s. 4d., respectively, and his effects amount to £870 16s. $10\frac{1}{2}$d. What will each receive?

(9.) There are 1800 warp threads in a fabric, the pattern of the warp being 12 threads red, 8 blue, 16 white. What length of yarn of each colour is there in the fabric, the length of the fabric being 140 yards?

(10.) Three persons, A, B, and C, freighted a ship with 680 tuns of wine, of which A supplied 220 tuns, B, 194, and C, the rest: in a storm the seamen threw overboard 85 tuns. How much must each person sustain of the loss?

(11.) Standard silver contains 37 parts of pure silver, and 3 of copper. How much of each is there in $27\frac{1}{2}$s.; 1 lb. Troy being coined into 66 shillings?

(12.) A field containing 37 acres 2 roods 14 poles, is to be divided among three persons—A, B, C—in proportion to the values of their estates. Now if A's estate is worth £500 a year, B's £320, and C's £75, what quantity of land must each have?

(13.) A, B, and C engage in trade, and gain £521 8s. 2d: A puts in £250 for 6 months; B, £300 for 5 months; C, £400 for 12 months. Required each partner's share of the gain.

(14.) Standard gold consists of 22 parts pure gold, with 2 parts copper, and from a pound Troy of this metal are coined $46\frac{29}{40}$ sovereigns. Find the quantity of pure gold in 100 sovereigns.

(15.) Divide £64 3s. 4d. among 4 persons, in the ratio of the fractions $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{10}$.

(16.) A, B, C rent a field for £42 15s.: A puts in 10 sheep for 5 months; B, 12 sheep for 6 months; C, 7 sheep for 9 months. What must each pay of the rent?

(17.) A ship worth £6000, and insured for $\frac{2}{3}$ its value, is entirely wrecked; $\frac{1}{2}$ belonged to A, $\frac{1}{3}$ to B, and the rest to C. What were their respective losses?

(18.) The rent of a piece of land is £185 8s.: A feeds 8 cows on it for 9 months; B, 10 cows for $7\frac{1}{2}$ months; and C, 36 cows for $4\frac{1}{2}$ months. What rent should each pay?

(19.) A and B enter into partnership for a year, A with £200, and B with £160; after 4 months they admit C with £120; at the end of the year their gain is £150. What is each man's share of it?

(20.) A, B, and C enter into trade: A puts in £600, but at the end of 8 months he withdraws £200; B puts in £400, and at 6 months' end £200 more; C puts in £300, and at the end of 4 months £400 more; but at the end of 10 months he takes out £200. They clear £360. What is each man's share?

XXX.—PERCENTAGES.

Percentages are calculations made at a certain rate *per cent.*, that is, per hundred.

As abstract numbers, they are continually occurring in questions of **Statistics, Analysis**, &c.

They most frequently, however, occur in the form of allowances, or payments of so much money on every £100. In such cases the expressions 2 *per cent.*, $3\frac{1}{2}$ *per cent.*, 5 *per cent.*, &c., mean, respectively, £2, £3 10s, or £5, for every £100.

All Percentages are, in fact, examples in Simple Proportion, in which one of the given terms is 100, though they often admit of readier methods of calculation.

Percentages are known in business under various names, as, **Commission, Brokerage, Insurance, Interest, Discount**, &c.

1.—Commission and Brokerage.

Commission and **Brokerage** are percentages allowed to agents or brokers for buying or selling goods, shares, &c.

The Commission, &c., having the same ratio to the amount on which it is charged as the rate per cent. has to £100, its amount may be found by a Simple Proportion; thus,

Example.—What is the brokerage on £1575 at $\frac{1}{8}$ per cent.?

$$\pounds 100 : \pounds 1575 :: \pounds\tfrac{1}{8} : \pounds x$$

$$x = \frac{\overset{63}{\cancel{1575}}}{\underset{4}{\cancel{100}}} \times \frac{1}{8} = \frac{63}{32} = \overset{\pounds}{1}\ \overset{s.}{19}\ \overset{d.}{4\tfrac{1}{2}}.$$

Hence we derive the following rule:—

Rule.—Multiply the sum on which the allowance is to be made by the rate per cent., and divide by 100.

The rate per cent. can, of course, be expressed by Vulgar Fractions, or Decimals, and the calculation made accordingly.

Thus, 5 per cent. is $\frac{1}{20}$ or ·05 of any given sum; similarly,

4 per cent. $= \frac{1}{25} = {}$·04; | 2 per cent $= \frac{1}{50} = {}$·02;
$3\frac{1}{3}$ per cent. $= \frac{1}{30} = {}$·0$\dot{3}$; | $1\frac{1}{4}$ per cent $= \frac{1}{80} = {}$·0125;
$2\frac{1}{2}$ per cent. $= \frac{1}{40} = {}$·025; | 1 per cent $= \frac{1}{100} = {}$·01.

Sometimes the rate may be conveniently sub-divided into *aliquot parts* of 100, and worked after the method of Practice.

Thus, $7\frac{1}{2}$ per cent. might be calculated by taking 5 per cent as $\frac{1}{20}$ of the given sum, and then $2\frac{1}{2}$ per cent. as $\frac{1}{2}$ of 5 per cent.

Example.—What is the commission on the sale of goods to the value of £375 7s. 6d., at $2\frac{1}{2}$ per cent?

$$\begin{array}{rrr}
\pounds & s. & d. \\
375 & 7 & 6 \\
 & & 2\frac{1}{2} \\
\hline
750 & 15 & 0 \\
187 & 13 & 9 \\
\hline
9{,}38 & 8 & 9 \\
20 & & \\
\hline
7{,}68 & & \\
12 & & \\
\hline
8{,}25 & &
\end{array}$$

Or thus:

$$2\tfrac{1}{2} = \tfrac{1}{40}\overline{\big)\begin{array}{rrr} \pounds & s. & d. \\ 375 & 7 & 6 \\ \hline 9 & 7 & 8\frac{1}{4} \end{array}}$$

Or thus:

$$\begin{array}{r}
\pounds \\
375{\cdot}375 \\
{\cdot}025 \\
\hline
1876875 \\
750750 \\
\hline
9{,}384375 \\
20 \\
\hline
7{,}687500 \\
12 \\
\hline
8{,}2500
\end{array}$$

Ans. £9 7s. $8\frac{1}{4}$d.

Ex. 73.

(1.) How much in *the pound* are the following percentages:—50, 40, 33⅓, 25, 20, 10, 5, 2½, 1¼?

(2.) How much *per cent.* are the following allowances per £:—17s. 6d., 16s. 8d., 13s. 4d., 7s. 6d., 5s., 3s. 4d., 2s. 6d., 1s., 6d.?

(3.) How much *per cent.* are 4d., 3d., 2d., 1½d., 1d. in the shilling?

Find the commission on:—

	£	s.	d.			£	s.	d.	
(4.)	187	16	10	at 2½ per cent.	(5.)	578	10	6	at 2⅜ per cent.
(6.)	320	17	6	at 3¼ per cent.	(7.)	496	16	8	at 7⅜ per cent.

Find the brokerage on:—

	£	s.	d.			£	s.	d.	
(8.)	1242	15	0	at ⅛ per cent.	(9.)	1874	10	6	at ½ per cent.
(10.)	3197	13	4	at 1⅝ per cent.	(11.)	2576	12	6	at 1⅛ per cent.

Find the duty on:—

	£	s.	d.			£	s.	d.	
(12.)	784	14	6	at 12½ per cent.	(13.)	457	10	0	at 17¼ per cent.
(14.)	396	2	9	at 9⅞ per cent.	(15.)	556	13	4	at 11⅔ per cent.

2.—INSURANCE.

INSURANCE, or rather the *premium of insurance*, is a percentage paid in order to secure the repayment of the value of property, if lost or destroyed, or the payment of a sum of money under certain conditions of accident or death.

The legal document or contract by which the payment is secured is termed the *Policy of Insurance*, and is liable to a *stamp duty*.

The Premium of Insurance is calculated in the same way as Commission or Brokerage.

As £100 : the sum to be insured :: the rate per cent. : the premium.

It is sometimes required to find for *what amount* goods of a certain value should be insured, so as, *not only to recover their value* in case of loss, *but also the amount of premium paid.*

If £100 worth of goods be insured at 3 per cent., the owner will, in case of loss, receive £100; but he has spent £3; his real receipt will, therefore, be only £97.

Every £100 insured, thus, only covers goods to the value of £100 *less* the rate of insurance; hence the following rule:—

Rule.—As £100 — the rate : the value of the goods :: £100 : the amount to be insured.

Example I.—Find the premium of insurance on £5760 at 3¼ per cent.?

$$\begin{array}{r} \pounds \\ 5760 \\ 3\frac{1}{4} \\ \hline 17280 \\ 1440 \\ \hline 187{,}20 \\ 20 \\ \hline 4{,}00 \end{array}$$

$$\overset{\pounds}{100} : \overset{\pounds}{5760} :: \overset{\pounds}{3\tfrac{1}{4}} : \overset{\pounds}{x}$$

$$x = 5760 \times 3\tfrac{1}{4} \div 100 =$$

$$\frac{\overset{144}{\cancel{5760}} \times 13}{\cancel{4} \times 10\cancel{0}} = \frac{1872}{10} = \overset{\pounds}{187}\ \overset{s.}{4}$$

Example II.—What sum should be insured, at 3¼ per cent., on goods worth £5760, so as to recover, in case of loss, *both value and premium?*

$$\overset{\pounds}{96\tfrac{3}{4}} : \overset{\pounds}{5760} :: \overset{\pounds}{100} : \overset{\pounds}{x}$$

$$x = \frac{5760 \times 100 \times 4}{387} = \frac{230400}{387} = \overset{\pounds}{5953}\ \overset{s.}{9}\ \overset{d.}{9\tfrac{9}{43}}.$$

Ex. 74.

Find the premium of insurance on ;—

(1.) £230 at 42s. per cent.; £525 12s. 6d. at 2½ guineas per cent.

(2.) £675 at 42s. per cent.; £4750 at 3 guineas per cent.

Find the expense of insuring;—

(3) £1757 at 42s. per cent.; policy duty 5s. 6d. per cent.*

(4.) £1270 at 21s. per cent.; policy duty 3s. 6d. per cent.

(5.) £1250 at £1 17s. 6d. per cent.; policy duty 2s. per cent.

(6.) £4850 at 15s. per cent.; policy duty 3s. 4d. per cent.

(7.) What will be the expense of insuring a cargo valued at £1050 15s.; premium 2 guineas per cent.; duty 3s. per cent.; agent's commission ½ per cent.?

---

[*] The Duty on Policies of Insurance is always charged on *exact hundreds, that is for any part* of £100 the same as for £100.

(8.) Find the expense of insuring a cargo worth £2550 at £2 12s. 6d. per cent.; duty 5s. per cent.; commission ⅛ per cent.

(9.) How much will it cost to insure a cargo worth £4530; premium £4 14s. 6d. per cent.; duty 4s. per cent.; commission ½ per cent.?

(10.) What will be the expense of insuring £1880 worth of property at £5 5s. per cent.; duty 5s. per cent.; commission ⅛ per cent.?

(11.) How much must be insured to cover the value and expense of insuring goods worth £734 8s. at 4 per cent.?

(12.) How much must be insured to cover the value and expense of insuring goods worth £427 13s. 4d.; premium 4⅝ per cent.; duty 5s. 6d. per cent.?

(13.) How much must be insured to cover the value of the goods and the *whole* expenses incurred in each of Nos. 7, 8, 9, 10?

3.—INTEREST.

INTEREST is the *percentage* allowed for the use of a sum of money for a certain time.

The sum lent is termed the **Principal.**

The **Rate per cent.** is the Interest of £100 for *one* year.

The Principal *and* Interest *together* are termed the **Amount.**

Interest is calculated like Commission, Brokerage, or any other percentage, only differing from them by the introduction of the idea of *time.*

RULE.—*As* £100 *is to* the given principal *so is* the rate per cent. (or interest on £100 for *one* year) *to* the interest on the given principal for *one* year. Then multiplying the interest so found by the given time, will give the interest sought.

It will be seen that this rule is but an application of Compound Proportion, thus:—

$$\left.\begin{array}{l}\pounds100 : \text{given principal} \\ 1 \text{ year} : \text{given time}\end{array}\right\} :: \text{rate per cent.} : \text{interest required.}$$

$$\text{Interest} = \frac{P \times T \times R}{100}$$

From this proportion we obtain the following practical rule fo
ing the interest :—

**RULE.—Multiply the principal by the rate per cent
time, and divide by 100.**

The calculation may of course be made in different ways.

EXAMPLE.—Find the interest on £1075 13s. 4d. for 3 year
per cent.

$$\begin{array}{rrr}
£ & \text{s.} & \text{d.} \\
1075 & 13 & 4 \\
 & & 2\frac{1}{2} \\ \hline
2151 & 6 & 8 \\
537 & 16 & 8 \\ \hline
2689 & 3 & 4 \\
 & & 3 \\ \hline
80{,}67 & 10 & 0 \\
20 & & \\ \hline
13{,}50 & & \\
12 & & \\ \hline
6{,}00 & &
\end{array}
\qquad
\begin{array}{c|c|rrr}
 & & £ & \text{s.} & \text{d.} \\
2\frac{1}{2} & \frac{1}{40} & 1075 & 13 & 4 \\ \cline{3-5}
 & & 26 & 17 & 10 \\
 & & & & 3 \\ \cline{3-5}
 & & £80 & 13 & 6
\end{array}$$

Or thus, $2\frac{1}{2} \times 3 = 7\frac{1}{2}$

$$\begin{array}{c|c|rrr}
 & & £ & \text{s.} & \text{d.} \\
5 & \frac{1}{20} & 1075 & 13 & 4 \\ \cline{3-5}
2\frac{1}{2} & \frac{1}{2} & 503 & 15 & 8 \\
 & & 26 & 17 & 10 \\ \cline{3-5}
 & & £80 & 13 & 6
\end{array}$$

Or thus :—

$$\text{Interest} = \overset{£}{1075\tfrac{2}{3}} \times 2\tfrac{1}{2} \times 3 \div 100 =$$

$$\overset{£}{\frac{3227}{\cancel{3}}} \times \frac{\cancel{5}}{2} \times \frac{\cancel{3}}{1} \times \frac{1}{\underset{20}{\cancel{100}}} = \overset{£}{\frac{3227}{40}} = \overset{£\ \ \text{s.}\ \ \text{d.}}{80\ \ 13\ \ 6}$$

Or thus :—

$$\text{Interest} = \overset{£}{1075.\dot{6}} \times .025 \times 3 = \overset{£}{80.674\dot{9}} = \overset{£\ \ \text{s.}\ \ \text{d.}}{80\ \ 13\ \ 6}$$

Ex, 75.

Find *one* year's interest on :—

(1.) £640 at 3 per cent. (2.) £1295 11s. at 4 per cen
(3.) £1605 3s. at 3½ per cent. (4.) £1545 13s. 6d. at 4¼ pe
(5.) £567 5s. 6d. at 4½ per cent. (6.) £561 8s. 4d. at 5 per c

Find the interest on :—

(7.) £500 for 4 years at 4¼ per cent.
(8.) £750 for 3⅝ years at 3½ per cent.
(9.) £1087 15s. for 12 years at £2 2s. 6d. per cent.

(10.) £1605 15s. for 8 years, at 1¼ per cent.
(11.) £1129 6s. 8d. for 5 years at 1¾ per cent.
(12.) £1749 17s. 4d. for 10 years at 3¾ per cent.

Find the amount of
(13.) £1144 10s. for 5 years 5 months at 4½ per cent.
(14.) £769 13s. 3d. for 3 years 2 months at 3¾ per cent.
(15.) £1500 for 2 years 73 days at £4 17s. 6d. per cent.
(16.) £245 13s. 4d. for 146 days at 4¾ per cent.
(17.) £1075 13s. 4d. for 4 years at 2½ per cent.
(18.) £1210 9s. 6d. for 4 months 2 weeks 2 days at 3¼ per cent.
(19.) 4175 francs 30 centimes for 219 days at 5½ per cent.
(20.) 5837 fr. 45 c. for 3 years 4 months 10 days at 6½ per cent.

Find the interest on
(21.) £308 from January 9 to July 25, at $2\frac{5}{18}$ per cent.
(22.) £115 from May 26 to November 18 at 6⅝ per cent.
(23.) £740 from March 10 to July 14 at $3\frac{7}{12}$ per cent.
(24.) £1440 from March 30 to June 29 at 1½ per cent.
(25.) £1560 for 257 days at 3¾ per cent.
(26.) £1168 for 159 days at £3 7s. 6d. per cent.

Interest is frequently required to be calculated on Deposit accounts where the rate per cent. varies, also on Accounts Current, and Loans, &c., payable by instalments.

This may be most readily done by the following practical rule for finding the interest *for days.*

Rule.—Multiply the principal by the number of days and *double* the rate and divide by 73000.

$$\text{For} \left\{ \begin{array}{lcl} £100 & : & \text{principal} \\ 365 \text{ days} & : & \text{number of days} \end{array} \right\} :: \text{rate} : \text{interest}.$$

$$I = \frac{P \times \text{no. of days} \times \text{rate}}{36500} = \frac{P \times \text{no. of days} \times \text{double rate}}{73000}$$

The latter formula has the more convenient divisor.

Example.—Borrowed £500 February 2; paid £80 March 9; £115 May 15; £25 June 1; £280 August 14. What interest was due at 4 per cent.?

$$\begin{array}{rcrcr}
\pounds & & \text{days} & & \\
500 & \times & 35 & = & 17500 \\
420 & \times & 67 & = & 28140 \\
305 & \times & 17 & = & 5185 \\
280 & \times & 74 & = & \underline{20720} \\
 & & & & 71545
\end{array}$$

The sums of money are the successive balances due at each date of payment.

The multipliers are the number of days between one payment and another.

$$I = \frac{71545 \times 8}{73000} = \frac{56236\cancel{0}}{7300\cancel{0}} = \pounds 7.703 = \pounds 7\ 14s.\ 1d.\ \text{nearly}.$$

Ex. 76.

(1.) Find the interest due on an account for £1000, due March 13, of which £300 were paid April 7; £234 May 13; £230 July 28; and the balance October 18, at 4¾ per cent.

(2.) Received a bill for £900, due January 1, of which £150 were paid February 28; £270 March 30; £173 June 19; £213 July 28; £57 September 23; and the balance November 17. Find the interest due at 3¾ per cent.

(3.) Borrowed March 20, 1864, £1000, of which I paid £300 September 17; £150 December 21; £220 February 23, 1865; and the balance July 23. What interest is due at 4 per cent.?

(4.) Borrowed £2500, Midsummer, 1860; repaid it by quarterly instalments of £250. Required the interest due at 5 per cent.

(5.) Advanced £5000 January 1, 1862; received £300 every 4 months. What interest is due at 3½ per cent., and what balance remained unpaid December 31, 1865?

(6.) What interest is due at 3 per cent. on the following account current?

Dr.					*Cr.*
June 8	To Balance	£156	August 7	By Cash	£96
July 7	„ Cash	85	Sept. 10	„ ditto	37
Sept. 13	„ ditto	77	October 18	„ ditto	71
Dec. 31	„ ditto	112	Nov. 29	„ ditto	106

(7.) Deposited in the Union Bank of London, February 7, £130, interest at 5 per cent. April 2 interest fell to 4 per cent.; May 10, it was 4½ per cent.; June 15, 5 per cent.; July 3, 5½ per cent.; August 31, 6 per cent. What interest was due on September 10?

(8.) Find the interest on £750 from August 1 to October 7, at 2½ per cent.; to May 15, at 3⅓ per cent.; and to July 10, at 4 per cent.

(9.) Find the interest on 4050 francs from January 7 to February 19, at 2½ per cent., to March 29, at 3 per cent.; to June 4, at 3¼ per cent.; to August 11, at 2¾ per cent., and to October 18, at 3 per cent.

(10.) What interest is due on the following account, to April 30, 1865, allowing A. B. 5 per cent., and C. D. 3¾ ditto?

Dr. A. B.'s account current with C. D. *Cr.*

March 8	To Balance	£80	June 18	By Cash	£172
July 4	„ Cash	74	Sept. 23	„ ditto	98
Oct. 29	„ ditto	108	Nov. 15	„ ditto	148
Dec. 17	„ ditto	217	Feb. 28	„ ditto	150
Jan. 18	„ ditto	135			

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1. *To find* Principal, Rate, *or* Time *when the* Interest *is given.*

The preceding examples are calculations of Interest when Principal, Time, and Rate are given; but from the compound proportion given above (page 145) when *any* three of these are given the fourth may be found; thus:—

$$\left.\begin{array}{lcl} \text{£}100 & : & \text{Principal} \\ 1 \text{ year} & : & \text{Time} \end{array}\right\} :: \text{Rate per cent.} : \text{Interest given.}$$

$$I = \frac{P \times R \times T}{100}; \qquad \text{(i.) } R = \frac{I \times 100}{P \times T}$$

$$\text{(ii.) } T = \frac{I \times 100}{P \times R}; \qquad \text{(iii.) } P = \frac{I \times 100}{R \times T}$$

2. *When the* Amount *is given to find the* Principal.

As the **Amount of £100** at the given rate and time *is to* the **given amount** *so is* **£100** *to* the **Principal** sought.

$$\textit{Or, } \text{(iv.) } P = \frac{100 \times A}{100 \times (R \times T)}$$

EXAMPLE I.—If 1240 gain £102 6s. interest in 1 year and 10 months, what is the *rate* per cent.?

$$\text{(i.)} \left.\begin{array}{ccc} \overset{\text{£}}{100} & : & \overset{\text{£}}{1240} \\ \overset{\text{year}}{1} & : & \overset{\text{years}}{1\frac{5}{6}} \end{array}\right\} :: \overset{\text{£}}{x} : \overset{\text{£}}{102\frac{3}{10}}$$

$$x = \frac{100 \times 102\frac{3}{10}}{1240 \times 1\frac{5}{6}} = \frac{\cancel{100} \times \overset{\overset{3}{\cancel{93}}}{\cancel{1023}} \times \overset{3}{\cancel{6}}}{\underset{\underset{2}{\cancel{62}}}{\cancel{1240}} \times \cancel{10} \times \cancel{11}} = \frac{9}{2} = 4\frac{1}{2} \text{ per cent.}$$

EXAMPLE II.—In what *time* will £725 amount to £813 15s. at 5 per cent.?

$$\text{(ii.)}\quad \left.\begin{array}{ccc} \overset{\pounds}{100} & : & \overset{\pounds}{725} \\ \overset{\text{year}}{1} & : & \overset{\text{years}}{x} \end{array}\right\} :: \overset{\pounds}{5} : \overset{\pounds}{108\tfrac{3}{4}}$$

$$x = \frac{100 \times 108\frac{3}{4}}{725 \times 5} = \frac{\overset{4}{\cancel{100}} \times \overset{87}{\cancel{435}}}{\underset{29}{\cancel{725}} \times \cancel{5} \times \cancel{4}} = \frac{87}{29} = 3 \text{ years.}$$

EXAMPLE III.—What *principal* will give £3 17s. 4d. interest in 58 days at 4 per cent.?

$$\text{(iii.)}\quad \left.\begin{array}{ccc} \overset{\pounds}{100} & : & \overset{\pounds}{x} \\ \overset{\text{days}}{365} & : & \overset{\text{days}}{58} \end{array}\right\} :: \overset{\pounds}{4} : \overset{\pounds}{3\tfrac{13}{15}}$$

$$x = \frac{100 \times 3\frac{13}{15}}{4 \times \frac{58}{365}} = \frac{\overset{\overset{5}{\cancel{25}}}{\cancel{100}} \times \cancel{58} \times 365}{\cancel{4} \times \underset{3}{\cancel{15}} \times \cancel{58}} = \frac{1825}{3} = \pounds 608 \text{ 6s. 8d.}$$

EXAMPLE IV.—What sum of money or *principal* put out to interest will amount to £3376 8s. 11½d. in 9 years and 7 months at £4 11s. 8d. per cent.?

(iv.) $\pounds 4\frac{7}{12} \times 9\frac{7}{12} = \pounds 43\frac{133}{144}$ = interest on £100 for $9\frac{7}{12}$ years.

$\therefore\ \pounds 143\frac{133}{144}$ = the amount of £100 for $9\frac{7}{12}$ years.

Then $\pounds 143\frac{133}{144} : \pounds 3376\frac{43}{96} :: 100 : x$

$$x = \frac{\overset{4}{\cancel{100}} \times 324139 \times \overset{6}{\cancel{144}}}{\underset{\cancel{24}}{\cancel{96}} \times \underset{829}{\cancel{20725}}} = \frac{1944834}{829} = \pounds 2346.$$

Ex. 77.

(1.) In what time will £750 amount to £812 10s. at 5 per cent.?

(2.) In what time will £802 17s. 6d. amount to £883 3s. 3d. at 2½ per cent.?

(3.) At what rate will £567 5s 6d. gain £178 13s. 9.99d. interest in 4½ years?

(4.) What principal put to interest at 5 per cent. will amount to £383 15s. 6d. in 3 months?

(5.) At what rate per cent. will £365 amount to £496 8s. in 8 years?

(6.) In what time will £262 10s. amount to £396 3s. 2d. at 3½ per cent.?

(7.) How much money must be put out to interest at 2½ per cent. *to gain* £170 2s. in 7 years 73 days?

) The interest on £15000 from May 4 to October 26 is
14s. 10$\frac{3}{7}\frac{6}{8}$d. what is the rate per cent.?

.) What principal will amount to £118 18s. 9d. in 7 years at 4¼
cent.?

0.) In what time will £1923 15s. gain £173 2s. 9d. at 3⅝ per cent.?

4.—COMPOUND INTEREST.

HEN the interest on a sum of money is added to the incipal at the end of a given term, and the sum so ineased becomes the Principal for the next term, the riginal sum is said to be at **Compound Interest.**

RULE.—Find the Amount for the first period as in Simple Interest, consider the first amount as the Principal for the second period, the second amount as the Principal for the third period, and so on.

The **Compound Interest** will be the sum of the successive interests; or it may be more readily obtained by subtracting the *original Principal* from the *last Amount.*

It will generally be found most convenient to work by Aliquot parts and Decimals, retaining *four* decimal places.

EXAMPLE.—Find the Compound Interest on £700 for 3 years at 5 per cent. payable (i.) yearly, (ii.) half yearly.

£5 per cent. per annum = £$\frac{1}{20}$ for a year, and £$\frac{1}{40}$ for half-a-year, hence:—

(i.)

		£
5	$\frac{1}{20}$	700
		35
5	$\frac{1}{20}$	735
		36.75
5	$\frac{1}{20}$	771.75
		38.5875
		810.3375
		700
		£110.3375

(ii.)

		£
5	$\frac{1}{40}$	700
		17.5
5	$\frac{1}{40}$	717.5
		17.9375
5	$\frac{1}{40}$	735.4375
		18.3859
5	$\frac{1}{40}$	753.8234
		18.8455
5	$\frac{1}{40}$	772.6689
		19.3142
5	$\frac{1}{40}$	791.9831
		19.7995
		811.7826
		700
		£111.7826

	£	s.	d.
Payable ½ yearly	111	15	8
„ yearly	110	6	9
Difference	1	8	11

Another mode of finding the amount of any sum of money at Compound Interest is as follows:—

Rule.—Multiply the **amount of £1** at the given **rate** for *one* period of time by itself as many times as there are periods, the result will be the *amount of* £1 for the given time.

Multiply this amount of £1 by the given number of pounds, and the product will be the **amount of the whole** at compound interest.

Example.—Find the amount of £1150 for 3 years, at 3 per cent., compound interest.

$1.03 \times 1.03 \times 1.03 \times 1150 = £1256.636 = £1256$ 12s. $8\frac{1}{2}$d.

The reason of the Rule may be shewn as follows:—

Amount of £1 for 1 year at 3 per cent. = £1.03.

Then £1 : £1150 : : £1.03 : 1st year's amount.
∴ 1st year's amount = £1150 × 1.03.

Again £1 : £1150 × 1.03 : : £1.03 : 2nd year's amount.
∴ 2nd year's amount = £1150 × 1.03 × 1.03.

Similarly £1 : £1150 × 1.03 : : £1.03 : 3rd year's amount.
∴ 3rd year's amount = £1150 × 1.03 × 1.03 × 1.03.
$= £1150 \times (1.03)^3$.
= £1256.636 = £1256 12s. $8\frac{1}{2}$d.

In the same way it may be seen that the

Amount for n years $= £1150 \times (1.03)^n$.

And the general formula for the Amount of any sum for n periods at Compound Interest will be:—

$$A = P \times \left(1 + \frac{r}{100}\right)^n$$

Ex. 78.

Find the Amount at Compound Interest of:—

(1.) £1440 for 3 years at 5 per cent.

(2.) £3125 for 3 years at 4 per cent

(3.) £550 for 6 years at 5 per cent.

(4.) £1866 13s. 4d. for 3 years at $2\frac{1}{2}$ per cent.

£875 for 15 months at 4 per cent. per annum, payable *quarterly*.
£364 for 3 years at 3¼ per cent.

the Compound Interest on :—
£750 16s. 8d. for 4 years at 4½ per cent.
£250 for 2½ years at 4 per cent. per annum, payable *half yearly*.
£300 for 1¾ years at 3 per cent. per annum, payable *quarterly*.
£764 12s. 6d. for 4 years at 5 per cent., payable *half yearly*.

A legacy of £1000 is left to a lad, aged 11 years, to accumulate cent. compound interest till he comes of age. What will it nount to?

The sum of £232 7s. 6d. has been left 8 years in a savings' what will it have accumulated to, at 3 per cent.?

5.—DISCOUNT.

COUNT is a per centage allowed for the payment of a f money before it is *legally* due.

PRESENT VALUE, or WORTH, of a sum of money le in the future is the sum so payable *less* the unt.

person holds a Bill of Exchange or Promissory Note, securing the payment of £100 at the end of 3 months, but wishes to get it *at once*, a banker (supposing the parties to be persons of will discount it, that is, will pay the £100 less the interest on months. Thus, if the rate of interest at the time were 5 per he holder of the bill would receive £100 — ¼ of £5, that is . The £1 5s. deducted is called the **Discount**, and the £98 15s. *ominal* **Present Value**.

ker's Discount, as usually charged, is simply the st on the *Future Value*, or full amount of the debt.

e Discount is the interest on the *True Present Value* debt.

True Present Value of a sum of money payable in ture is the *Principal, which put to interest at the given ould amount to that sum in the given time.*

e the finding of the true present value is the finding of the al when the rate, time and amount are given (page 149).

RULE.—*As* the **Amount** (or Future Value) **of £100** at the given rate and time *is to* the **whole Amount** (or Future Value) **of the debt,** *so is* **£100** (the Present Value of £105 payable in the future) *to* the **Present Value of the debt.**

EXAMPLE.—Find the true Present value of the above bill for £100 due 3 months hence, at 5 per cent.

$$\begin{array}{ccccccc} \text{F. V.} & : & \text{F. V.} & :: & \text{P. V.} & & \text{P. V.} \\ 101\frac{1}{4} & : & 100 & :: & 100 & : & x \end{array}$$

$$x = \frac{\overset{20}{\cancel{100}} \times 100 \times 4}{\underset{81}{\cancel{405}}} = \frac{8000}{81} = \text{£}98\ 15\text{s.}\ 3\tfrac{19}{27}\text{d.}$$

The True Discount = £100 — £98 15s. $3\frac{19}{27}$d. = £1 4s. $8\frac{8}{27}$d.

Banker's Discount is the True Discount *plus* the **Interest on the True Discount.**

Thus, in the foregoing example, 3 months' interest at 5 per cent. on the true Discount added to the true Discount, will give the Banker's Discount stated above.

$$\text{£}1\ 4\text{s.}\ 8\tfrac{8}{27}\text{d.} \times 5 \times \tfrac{1}{4} \times \tfrac{1}{100} = \frac{\overset{20}{\cancel{8000}} \times 5}{27 \times \cancel{4} \times \cancel{100}} = \tfrac{100}{27}\text{d.} = 3\tfrac{19}{27}\text{d.}$$

	£	s.	d.
True Discount ...	1	4	$8\frac{8}{27}$
Interest on ditto ...	0	0	$3\frac{19}{27}$
Banker's Discount	1	5	0

TRADESMAN'S DISCOUNT is simply a percentage allowed for ready money payments, without reference to time.

Thus, a bookseller often allows his customer an abatement of 2d. on the shilling, or $16\frac{2}{3}$ per cent.

Ex. 79.

(1.) What is the *true* discount on a bill for £676 13s. 4d. due in 6 months at 3 per cent.?

(2.) What is the present value of £1183 4s. 8d. due in 15 months at 8 per cent.?

(3.) Find the present worth of £710 10s due 4 months hence at $4\frac{1}{2}$ *per cent*

(4.) Find the *true* discount on £1000 due four years hence, at 5 per cent.

(5.) Find the *true* discount on a bill for £750, which has 9 months to run, at 4 per cent.

(6.) What is the present worth of £256 7s. 6d. due 3 years hence, at 3 per cent. ?

(7.) What is the *true* discount on a bill of £324 16s. 7½d., due 2 years and 9 months hence, at 3½ per cent. ?

(8.) What is the *true* discount on £1000 10s , due 5 years 4 months hence, at 4½ per cent. ?

(9.) Find the present worth of £594 14s. 9d., due 8 months hence, at 5¾ per cent.

(10.) What is the difference between the *true* discount and the *banker's* discount on £126 13s. 4d., due in 4 months, at 8 per cent. ?

(11.) What is the present worth of a bill for £1875, due in 79 days at 5 per cent. ?

(12.) Find the *true* discount on £598 9s. 9d. due in 1 year 115 days at 2½ per cent.

(13.) Find the present worth of 1175 francs 50 centimes, due 73 days hence, at 4⅕ per cent. ?

(14.) What will be the difference between the interest and *true* discount on 12770 francs, due in 2¼ years, at 5½ per cent. ?

(15.) Find both the common and true discount on the following bills :—

		drawn			discounted	
(i.)	£218 11s. 8d.	Aug. 14	at 4	months,	Oct. 3,	at 4 per cent.
(ii.)	£568 12s. 9d.	April 27	,, 7	,,	June 3	,, 5 per cent.
(iii.)	£1570 10s. 6d.	Dec. 30	,, 5	,,	Jan. 10	,, 3½ per cent.
(iv.)	£157 15s. 0d.	April 30	,, 3	,,	May 30	,, 3½ per cent.

6.—STOCKS, SHARES, &c.

STOCK is the name given to the money or capital of a trading company.

GOVERNMENT STOCKS consist of moneys lent either to the British or other governments, on which interest is payable at certain fixed rates.

Government Stocks are often called "**The Funds**," the interest being paid out of the public funds, or moneys received from the assessed taxes, customs, stamp, and excise duties, &c.

Government Stocks are mostly named according to the annual rate of interest payable, thus we have, *Three per Cents.*, *India Five per Cents.*, *Turkish Six per Cents.*, &c.

In ordinary cases, when one person lends money to another, he expects to receive it again after a fixed time, but when a Government borrows money, or, as it is termed, *effects a Loan*, it does not pledge itself to repay what it borrows until it finds it convenient, but it only pledges itself to pay regularly, in the meanwhile, the interest agreed on.

Though the Government does not agree to repay the lenders the sums of money lent, it agrees that the lenders may dispose of or *transfer* their claims for the interest thereon to whom they please.

The amount that can be obtained for £100 of Stock is called the **Price** of the Stock.

Thus, when it is said that the 3 per cents. are at 91, it means that £100 stock, on which £3 are payable annually, can be purchased for £91.

The prices of Stocks or Shares rise and fall, according to the changes in the value of money, the varying rates of interest obtainable on other securities, the probable success or otherwise of a company, the prospects of peace or war, &c., &c.

In the case of trading companies the profits are usually divided half-yearly among the shareholders, who can transfer their shares in the same way as the holders of Government Stock.

These half-yearly payments are called **Dividends.**

When £100 stock or share will sell for *exactly* £100 cash, the stock is said to be at **par.**

When it will sell for *more* than £100 cash, it is said to be at so much **premium.**

When it will sell for *less* than £100 cash, it is said to be so much **discount.**

Since the interest paid by government is considered perfectly sure of being regularly paid, it is generally less than can be obtained on other securities, and, therefore, government stock is usually at a discount.

For no one would pay £100 cash for a right to receive £3 per annum, when he could obtain £5 elsewhere *with equal security.*

The *price* of a stock or share is its **actual value** in cash.

The *amount* of a stock or share is its **nominal value** on which the interest is paid.

Stocks and shares are bought or sold through agents called brokers, who receive a percentage for their trouble; in the case of the British Government stocks this brokerage is $\frac{1}{8}$ per cent.

Various questions may be asked about stocks and shares, for instance —what is the cash value of so much stock?—what is the income arising from it?—what is the real rate of interest received?

All such calculations referring to Stocks and Shares are applications of the rules of proportion, and may be worked accordingly.

Example.—(i.) How much 3 per cent. stock at 93 can be bought for £7533 cash? (ii.) what yearly income will be obtained from it? (iii) what is the real rate of interest obtained?

(i.) Since £93 *cash* will purchase £100 stock, the amount of stock purchased will be to £100 stock, in the ratio of 93 to 7533 :—

$$\begin{array}{cccccccc} & \text{cash} & : & \text{cash} & :: & \text{stock} & : & \text{stock} \\ & £ & & £ & & £ & & £ \\ \therefore & 93 & : & 7533 & :: & 100 & : & x \end{array}$$

$$x = \frac{\overset{2511}{\cancel{7533}} \times 100}{\underset{31}{\cancel{93}}} = \frac{251100}{31} = £8100 \text{ Stock.}$$

(ii.) The yearly income will be the interest on £8100 stock at 3 per cent

$$\text{Income} = \frac{81\cancel{00} \times 3}{\cancel{100}} = £243.$$

(iii.) Here a *nominal* £100, or £93 *cash*, gains £3 *interest*, w interest will £100 *cash* gain?

$$\begin{array}{ccccccc} \text{cash} & : & \text{cash} & :: & \text{interest} & : & \text{interest} \\ £ & & £ & & £ & & £ \\ 93 & : & 100 & :: & 3 & : & x \end{array}$$

$$x = \frac{100 \times \cancel{3}}{\underset{31}{\cancel{93}}} = \frac{100}{31} = £3\tfrac{7}{31} \text{ *real rate* of interest.}$$

EXAMPLE II.—What will £2600 3 per cent. stock realise when i at 90⅜ per cent.; brokerage ⅛ per cent.?

Here each £100 *stock* will fetch £90⅜ — ⅛ = 90¼ *cash*.

$$\begin{array}{cccccccc} & \text{stock} & : & £\ \text{stock} & :: & \text{cash} & : & \text{cash} \\ & £ & & £ & & £ & & £ \\ \therefore & 100 & : & 2600 & :: & 90\tfrac{1}{4} & : & x \end{array}$$

$$x = \frac{2600 \times 90\frac{1}{4}}{100} = \frac{\overset{13}{\cancel{2600}} \times 361}{\cancel{100} \times \underset{2}{\cancel{4}}} = \frac{4693}{2} = £2346\ 10\text{s.}$$

EXAMPLE III.—A person transfers £2000 *stock* from the 3 per c at 90 to the India Five per cents. at 105. Find the alteration in income.

$$\text{His present income} = \frac{20\cancel{00} \times 3}{\cancel{100}} = £60.$$

Then the new stock being dearer he will have less for his money.

$$\begin{array}{cccccccc} & \text{price} & : & \text{price} & :: & \text{stock} & : & \text{stock} \\ & £ & & £ & & £ & & £ \\ \therefore & 105 & : & 90 & :: & 2000 & : & x \end{array}$$

$$x = \frac{\overset{6}{\cancel{90}} \times 2000}{\underset{7}{\cancel{105}}} = \frac{12000}{7} = £1714\tfrac{2}{7} \text{ India Stock.}$$

£1714$\tfrac{2}{7}$ India Stock at 5 per cent. = new income.

$$\text{But } \frac{1200\cancel{0}}{7} \times \frac{5}{1\cancel{00}} = \frac{600}{7} = £85\tfrac{5}{7}.$$

$\therefore$ Alteration in income = £85$\tfrac{5}{7}$ — £60 = £25$\tfrac{5}{7}$ *gain*.

Ex. 80.

Find the annual **income** from the following amounts of stock:—

(1.) £750 stock in the 3 per cent. Consols.

(2.) £17862 10s. in the India 5 per cents.

(3.) £82304 15s. in the New 3 per cents.

652 13s. 4d. in the Chilian 4½ per cents.
361 12s. 6d. in the 3 per cent. Reduced.
961 2s. 6d. in the Danubian Seven per cents.

e **value** of the following amounts of stock :—
500 stock at 86⅛. (8.) £375 stock at 97½.
1320 stock at 93¾. (10.) £7334 11s. 8d. at 94¼.
956 16s. stock at 78¼. (12.) £4311 8s. 9d. at 85⅜.

e **cost** of the following amounts of stock, including ⅛ per
kerage : —
1060 10s. at 93¼. (14.) £556 15s. at 97⅛.
20000 at 82⅝. (16.) £1378 17s. 6d. at 102¾.

he **selling price** of the following amounts of stock, including
t. brokerage :—
330 at 87½. (18.) £5050 at 77¼.
2736 10s. 6d. at 90⅜. (20.) £1254 18s. 8d. at 95⅝.

A person buys £1250 stock at 98¾ and sells it at 103, how
es he gain ?

If I invest £1700 *cash* in the 3 per cents. at 93¼, what will be
yearly dividend ?

How much *cash* must be invested in the 3½ per cents. at 93, to
an income of 1000 guineas a year ?

How much *stock* must be sold out at 96⅛ to pay a debt of
6d., and brokerage ⅛ per cent. ?

What is the real rate of interest when the New 3 per cents.
⅝ ?

At what price must £1000, 3½ per cent. stock be bought,
⅛ per cent. brokerage, to return 5 per cent. on the money
?

Find the difference of income arising from selling £1100, 3 per
ck at 87¼, and buying £100 railway shares, paying 5½ per
£112 ?

If I invest £8725 in 3 per cent. Consols, at 87⅜ ; then sell
90¾, and with the proceeds buy 4 per cents. at 105¼ ; allowing
nt. brokerage on each transaction, what will be my annual
om the last named stock ?

What will be the income on the transfer of £2000 3 per cent.
72 per cent., to the 4 per cents., at 91⅛ per cent., with a

brokerage of $\frac{1}{8}$ per cent. on each transaction? and what will be the difference of the dividends?

(30) Which is the better investment in the French Funds, the 3 per cents. at 68 fr. 95 c., or the $4\frac{1}{2}$ per cents. at 97 f. 26 c.? What income would result from investing 100,000 francs in each stock?

(31.) At what price, in francs, ought the French $4\frac{1}{2}$ per cents. to be, to return 5 per cent. on the money invested? And how much must be invested to give an income of 10,000 francs a year?

7.—STATISTICS.

STATISTICS is the science which is concerned in collecting and arranging facts illustrative of the condition and resources of a *state* or country,—its population, revenue, trade and commerce, the employments of the people, the state of education, &c., &c.

Statistical statements are usually arranged in tabular form, and the numerical results stated as **percentages.**

All such statistical problems are questions in Proportion, one of the given terms being 100, and the result should be expressed in decimals.

EXAMPLE I.—A government office expended £2763 in 1864, and £3186 15s. in 1865. Find the increase per cent.

$$\begin{array}{rl} \pounds & \\ 3186.75 & \text{Expenditure in } \mathbf{1865.} \\ \underline{2763} & \text{,, \quad ,, \quad } \mathbf{1864} \\ 423.75 & \text{Actual increase.} \end{array}$$

$$\begin{array}{ccccccc} \pounds & & \pounds & & \pounds & & \pounds \\ 2763 & : & 100 & :: & 423.75 & : & x \end{array}$$

$$x = \frac{423.75 \times 100}{2763} = \frac{42375}{2763} = 15.34 \text{ per cent.}$$

EXAMPLE II.—The number of marriages during 1863 in a certain district was 19639; in 1864 the actual number had increased 4.318 per cent. Find the number in the latter year.

$$\text{Here } 100 : 104.318 :: 19639 : x.$$

$$x = \frac{19639 \times 104.318}{100} = 196{,}39 \times 104.318 = 20487.$$

Ex. 81.

(1.) The year's revenue from the tax on railways to March 31st, 1864, was £403,625 8s., and to March 31st, 1865, it was £439,332 1s. What was the increase per cent.?

(2.) The total income and expenditure of the United Kingdom for the years ending 31st March, 1864 and 1865, were as follows:—Find the increase or decrease *per cent.* on each head.

	1864.	1865.
Income	£73,455,429	£73,481,986
Expenditure	71,102,751	70,250,756

(3.) Find the actual surplus income of each year (in the last question), and the increase per cent. of the surplus of 1865 over that of 1864.

(4.) The population of England and Wales in 1861 was 20,061,725, of whom 9,758,852 were *males*. What percentage did each sex bear to the whole population?

(5.) How much per cent. had the population of London increased in 10 years? In 1851 it was 2,362,236, and in 1861 it was 2,803,034.

(6.) Find the percentage of increase or decrease in the populations given in Ex. 53 and 55, page 9.

(7.) The total value of British and Irish produce exported rose from £146,602,342 in 1863 to £160,436,302 in 1864. What was the increase per cent.?

(8.) The populations of 3 towns in 1851 were 20325, 41304, and 6117; in 1861 they had increased respectively 6, 10, and 13 per cent. What was the *average* population in 1861?

(9.) The number of railway travellers during a certain period were, First-class, 4,743,213; Second-class, 10,291,749; Third-class, 21,409,217. Find what percentage each class was of the whole number of passengers.

(10.) Find the increase or decrease *per cent.* on the following sources of revenue for the years ending 31st Dec., 1864 and 1865:—

	1865. £	1864. £
Customs............	21,707,000	22,535,000
Excise	19,649,000	19,343,000
Stamps	9,636,000	9,468,000
Taxes	3,364,000	3,261,000
Property Tax ...	7,603,000	7,999,000
Post Office.........	4,250,000	4,060,000
Crown Lands ...	314,000	307,500
Miscellaneous ...	2,673,478	3,151,874
Totals ...	69,196,478	70,125,374

XXXI.—PROFIT AND LOSS.

PROFIT AND LOSS includes all calculations involving the difference between the **Buying Price** or **Prime Cost** of goods, and their **Selling Price.**

The **Profit** or **Loss** may be expressed as so much *on the whole*, but more generally at so much *per cent.*

Questions in Profit and Loss are really examples in Proportion, but a little ingenuity is often required in stating them.

Questions involving the *total* gain or loss may evidently be solved by the method of Practice and Compound Subtraction.

Questions involving the *percentage* of gain or loss, may be solved by the method of Proportion like other percentages.

A few examples will best illustrate the mode of treating questions of this kind.

EXAMPLE I.—Bought 3 cwt. 1 qr. of cheese at £2 6s. 8d. per cwt., and sold it at 6½d. per lb. (i.) What was the actual gain or loss? (ii.) What was the gain or loss per cent.?

$$\begin{array}{llllrrrl} & \text{cwt.} & \text{qr.} & \text{lbs.} & \text{£} & \text{s.} & \text{d.} & \\ \text{(i.)} & 3 & 1 & \text{or } 336 \text{ at } 6\tfrac{1}{2}\text{d per lb.} = & 9 & 17 & 2 & \text{Selling Price} \\ & 3 & 1 & \text{at £2 6s. 8d. per cwt.} = & 7 & 11 & 8 & \text{Prime Cost} \\ \cline{5-7} & & & & 2 & 5 & 6 & \text{Total Gain.} \end{array}$$

(ii.) Here £7 11s. 8d. gains £2 5s. 6d., what then will £100 gain?

$$\begin{array}{ccccccccccc} \text{£} & \text{s.} & \text{d.} & & \text{£} & \text{s.} & \text{d.} & & \text{£} & & \text{£} \\ 7 & 11 & 8 & : & 2 & 5 & 6 & :: & 100 & : & x \end{array}$$

$$x = \frac{10\cancel{0} \times \overset{3}{\cancel{546}}}{\cancel{1820}} = £30$$

∴ rate of gain is 30 per cent.

EXAMPLE II.—At what rate per lb. must sugar bought at £2 10s. per cwt. be sold so as to gain 12 per cent.?

Here £100 is to become £112, what then must £2 10s. become?

$$\begin{array}{ccccccc} £ & & £ & & £ & & £ \\ 100 & : & 112 & :: & 2\frac{1}{2} & : & x \end{array}$$

$$x = \frac{\cancel{5} \times \overset{56}{\cancel{112}}}{\cancel{2} \times \underset{20}{\cancel{100}}} = \frac{56}{20} = £2\ 16s.\ 0d. \text{ per cwt.}$$

$$\begin{array}{ccc} £ & s. & d. \\ 2 & 16 & 0 \end{array} \div 112 = 6d. \text{ per lb.}$$

Example III.—If I sell goods for £52 10s. at a loss of 12½ per cent. on the cost price; what was that cost price?

Here, if £100 worth of goods sells for £100−£12½, *i. e.* for £87½, what worth of goods will sell for £52½?

$$\begin{array}{ccccccc} \text{selling price} & : & \text{selling price} & :: & \text{prime cost} & : & \text{prime cost} \\ £87\frac{1}{2} & : & £52\frac{1}{2} & :: & £100 & : & £x \end{array}$$

$$x = \frac{\overset{4}{\cancel{100}} \times \overset{15}{\cancel{105}} \times \cancel{2}}{\cancel{2} \times \underset{7}{\cancel{175}}} = £60 \text{ prime cost.}$$

Example IV.—Sold goods for £30 and *lost* 25 per cent. on my outlay; for how much should they have been sold to have *gained* 33 per cent.?

Here £100 realized only £75 instead of £133.

$$\begin{array}{cccccccc} & £ & & £ & & £ & & £ \\ \therefore \text{ as} & £75 & : & 133 & :: & 30 & : & x \end{array}$$

$$x = \frac{\overset{2}{\cancel{30}} \times 133}{\underset{5}{\cancel{75}}} = \frac{266}{5} = £53\ 4s.$$

Example V.—Sold goods with a gain of 16 per cent., and of £8 8s. 8d., in all. Required the prime cost and selling price.

$$\begin{array}{ccccccc} \text{gain} & : & \text{gain} & :: & \text{prime cost} & : & \text{prime cost} \\ £16 & : & £8\frac{13}{30} & :: & £100 & : & £x \end{array}$$

$$x = \frac{10\cancel{0} \times 253}{3\cancel{0} \times 16} = \frac{2530}{48} = £52\ 14s.\ 2d.$$

£	s.	d.	
52	14	2	prime cost.
8	8	8	gain.
61	2	10	selling price.

Example VI.—Bought goods for £90 with three months' credit; for how much must they be sold, the same day, to gain 5 per cent., and give 9 months' credit—interest at 5 per cent?

Here £100 would, at 5 per cent., become in 3 and 9 months respectively, £101¼, and £103¾.

And £100 is to become £105.

$$\left.\begin{array}{ccc} \pounds & & \pounds \\ 101\frac{1}{4} & : & 103\frac{3}{4} \\ 100 & : & 105 \end{array}\right\} :: \begin{array}{ccc} \pounds & & \pounds \\ 90 & : & x \end{array}$$

$$x = \frac{9\cancel{0} \times \overset{83}{\cancel{415}} \times \overset{7}{\cancel{105}} \times \cancel{4}}{\cancel{4} \times \underset{\underset{3}{\cancel{45}}}{\cancel{405}} \times \underset{2}{\cancel{100}}} = \frac{581}{6} = \pounds 96\ 16s.\ 8d.$$

Ex. 82.

(1.) Goods bought for £52 14s. 2d., were sold for £69 11s. 6d. What was (i.) the *total* gain, (ii.) and the gain per cent?

(2.) Bought 468 yards of cloth at 8s. 8d. per yard, and sold them at 13s. per yard. Required (i.) the total gain, (ii.) the gain per cent.

(3.) Sugar bought at £2 4s. 4d. per cwt., is sold at 2¼d. per lb. profit. What is (i.) the total gain on the sale of 3 cwt. 3 qrs. 16 lbs., (ii.) the selling price per lb.?

(4.) At what price must butter, which costs 10d. per lb., be sold to gain 15 per cent.?

(5.) If tea be bought at 4s. 8d. per lb., what must it be sold for to gain 12½ per cent.?

(6.) If sugar be bought at 64s. per cwt., and sold at 76s., what is the gain per cent.?

(7.) If cheese bought at £2 6s. 8d. per cwt., is sold at 6½ per lb, what is the gain per cent.?

(8.) Coffee bought at £9 6s. 8d. per cwt., is sold at a profit of 10 per cent., what is the selling price per lb.?

(9.) Goods bought for £143 6s. 8d. were sold for £114 13s. 4d. What was the loss per cent.?

(10.) Starch sold at 6d. per lb., realized a profit of 33⅓ per cent. What was the prime cost per cwt.?

(11.) Sold tea at 4s. 4d. per lb., and thereby lost 7¼ per cent., what was the prime cost?

(12.) Rice sold at 1½d. per lb., gained a profit of 20 per cent. What was the prime cost of 3 cwt. 3 qrs. 14 lbs.?

(13.) By selling paper at 8s. 3d. per ream, I lost 1 per cent., how should it have been sold to have gained 13 per cent. ?

(14.) A person sells a watch for £39 12s., and loses 12 per cent. ; (i.) what did the watch cost him, (ii.) what should he have sold it for to gain 15 per cent. ?

(15.) Bought goods at 10 guineas per cwt., sold them at half-a-crown per lb. What was the gain per cent.

(16.) If by selling butter at 105s. 6d. per cwt., I gain 5½ per cent. at what rate per lb. must I sell it to gain 40 per cent. ?

(17.) A merchant by selling silk at 8 francs per mètre lost 4 per cent. What should be the price per mètre to gain 20 per cent. ?

(18.) Sold cheese at a profit of 1½d. per lb., or 37½ per cent. on the prime cost. What were the cost price and selling price per cwt. ?

(19.) An edition of 1000 copies of a work is sold at a gain of 9d. a copy, or 25 per cent on the outlay. What was (i.) the prime cost of the whole edition ; (ii.) the selling price of each copy ?

(20.) Equal quantities of tea at 3s. 6d. and 4s. 6d. per lb. are mixed together, and sold at the rate of 4s. 4d. per lb. What is the gain per cent. ?

(21.) By retailing sugar, which cost 53s. 8d. per cwt. at 5d. per lb., a grocer lost £3 18s. 9d. How much sugar did he sell ?

(22.) A fruit-salesman bought 360 chests of oranges, and sold one-half at 30 per cent. profit, and the other half at 10 per cent. loss, but gained on the whole £27. Required the prime cost and selling prices per chest.

(23.) Bought goods for £50 and sold them the same day for 50 guineas with 6 months' credit (interest 5 per cent.). What was gained or lost per cent.?

(24.) Sold for £304 cash, 720 yards of cloth, which had been bought 4 months before at 12s. 6d. per yard (interest 7½ per cent.). How much did I gain or lose ?

(25.) What shall I gain or lose per cent. by selling tea at 5s. 4d. per lb. and 8 months' credit, interest 5 per cent., for which I paid £25 per cwt. *a year ago ?*

(26.) Bought paper at 12s. ; at what must I sell it, and give 6 months' credit, at 5 per cent., to gain 10 per cent. ?

XXXII.—EQUATION OF PAYMENTS, AVERAGES, &c.

Equation of Payments is the method of finding the *equated* or *average* time for the payment in *one sum* of two or more debts payable at different times.

As some of the money will be paid *before* and some *after* the time when it is actually due, the equated time should be such, that the interest on the money due before that period should be equal to the discount on the money due after it; in actual business, however, it is usual to make the equated time such that the interest on the money due before and after it shall be the same.

Now the several portions of interest will evidently be proportionate to the products of the numbers denoting the several debts and times, and the sum of these products must equal the product of the numbers representing the whole debt and the equated time, hence the following rule:—

Rule.—Multiply each debt by the number of days, weeks, or months, before it becomes due. Add the products together and divide the total by the whole amount of the debts, the quotient will be the equated time or number of days, weeks, or months hence, at which the whole debt should be paid.

Example.—Four sums of £100, £200, £300, and £400 are due respectively at the end of 1, 2, 3, and 4 months. Find the equated time for paying the whole at once.

$$\begin{array}{rcrcr}
100 & \times & 1 & = & 100 \\
200 & \times & 2 & = & 400 \\
300 & \times & 3 & = & 900 \\
\underline{400} & \times & 4 & = & \underline{1600} \\
1000 & \times & x & = & 3000
\end{array}$$

$$\text{Equated time} = x = \frac{3000}{1000} = 3 \text{ months.}$$

The average price or value of a compound formed of ingredients differing in quantity and value may be found in a similar way.

Example.—Find the price per gallon of the following mixture of spirits, 15 gallons at 10s. 6d., 27 at 12s. 6d., 18 at 16s. 6d., and 9 at 18s.

$$\begin{array}{rlll}
\text{gals.} & \text{s.} & \text{d.} & \text{s.} \\
15 \text{ at } & 10 & 6 & = 157\frac{1}{2} \\
27 \text{ at } & 12 & 6 & = 337\frac{1}{2} \\
18 \text{ at } & 16 & 6 & = 297 \\
9 \text{ at } & 18 & 0 & = 162 \\
\hline
69 \text{ at } & x\text{s.} & & = 954\text{s.}
\end{array}$$

$$\text{Average price} = x = \frac{954}{69} = 6\text{s. per gall.}$$

The several methods of solving questions treating of the prices and quantities of a compound and its ingredients form the rule termed Alligation; but such questions, as well as those included under a rule termed Position, are more readily solved by Algebraic methods and the exercise of a little common sense.

Ex. 83.

Find the equated time for paying in one sum the following debts:—

(1.) £400 in 6 months, £500 in 7 months, £200 in 8 months, and £100 in 9 months.

(2.) £380 payable to-day, £440 in 12 days, and £620 in 24 days.

(3.) £200 in 2 months, £100 in 3½ months, and £150 in 5 months.

(4.) £74 in 50 days, £108 in 250 days, £100 in a year.

(5.) £200 in ⅓ of a year, £300 in ⅔ of a year, £200 in 1 year, £200 in 1¼ year, £100 in 2 years.

(6.) £36 10s. in 3½ months, £48 12s. in 6⅓ months, and £100 in 8½ months.

(7.) £136 12s. 6d. in 75 days, £284 10s. 9d. in 66 days, and £335 16s. 8d. in 90 days.

(8.) ¼ of a debt in 3 months, ⅓ in 4 months, ⅕ in 5 months, ⅙ in 6 months, and the rest in 7 months.

(9.) $\frac{2}{7}$ of a debt in 2 months, $\frac{4}{7}$ in 4 months, and the remainder in 5 months.

Find the average price of the following mixtures:—

(10.) Tea, 10 lbs. at 3s. 4d., 7 lbs. at 3s. 9d., 7 lbs. at 4s. 2d., 9 lbs. at 4s. 6d.

(11.) Potatoes, 12 bushels at 2s., 9 at 3s., 9 at 3s. 6d.

(12.) Wine, 8 gallons at 12s., 2 at 15s., 1 at 18s., 4 at 19s., 4 at 21s.

(13.) Sugar, 27 lbs. at 4d., 34 lbs. at 4½d., 75 lbs. at 5d., 38 lbs. at 6d., and 26 lbs. at 6½d.

(14.) Wheat, 18 qrs. at 96s., 14 qrs. at 84s., 21 qrs. at 78s., 13 qrs. at 76s., 12 qrs. at 75s., 12 qrs. at 74s.

(15.) Tobacco, 112 lbs. at 3s. 9d., 224 lbs. at 4s. 3d., 56 lbs. at 4s. 6d.

(16.) Malt, 12 bushels at 4s., 24 at 5s., 36 at 6s, 12 at 6s. 6d.

(17.) Wine, 1 hectolit. 48 lit. at 1 fr. 10 cent. per litre, 75 litres at 90 c., 3 hectolit. at 1 fr. 25 c.

(18.) Coffee, 17 kilog. at 2 fr. 50 c., 23½ kilog. at 2 fr. 60 c., 25 kilog. at 2 fr. 75 c., 34½ kilog. at 2 fr. 90 c.

XXXIII.—BARTER, EXCHANGE, CHAIN RULE.

By Barter is meant the exchanging of one commodity for an equal value of another.

By Exchange is meant the exchanging of the money of one country for that of another.

The **Par of Exchange** is "the intrinsic comparative value of the money of two countries, estimated by the weight and fineness of the coins."

When a given sum in the current money of one country can be exchanged for a sum in the current money of another equal to it in *real* or *intrinsic value*, as metal, the exchange is said to be at **par**, that is *equal.*

The **Course of Exchange** is the variable sum of the money of one country which is equivalent to a certain sum in the money of another.

Agio is the difference per cent. in some foreign places between the value of the money used in wholesale transactions and bank business, and the current money of the country; the intrinsic value of the current money being sometimes much less than the nominal value.

The **Arbitration of Exchanges** is the calculating, by the rate of exchange at different places, what profit or loss will be made by transmitting money through one or more places to another place.

Instances of Barter are very seldom met with, but examples of Exchange are continually occurring in business.

Questions in Barter and Exchange may be solved by the principles of Proportion, or by what is termed the CHAIN RULE.

The **CHAIN RULE** is a compendious rule for working examples which involve several statings in Proportion.

EXAMPLE I.—If 3 lbs. of tea be worth 8 lbs. of coffee, and 5 lbs. of coffee be worth 18 lbs. of sugar, how many pounds of sugar are worth 20 lbs. of tea?

Here, by two simple statings, we may *first* find how much tea is worth 5 lbs. of coffee, or 18 lbs. of sugar; and *then*, how much sugar is worth 20 lbs. of tea.

$$\text{(1st.)}\quad \overset{\text{coffee}}{8\text{ lbs.}} : \overset{\text{coffee}}{5\text{ lbs.}} :: \overset{\text{tea}}{3\text{ lbs.}} : \overset{\text{tea}}{\frac{3\times5}{8}\text{ lbs.}}$$

$$\text{(2nd.)}\quad \overset{\text{tea}}{\frac{3\times5}{8}\text{ lbs.}} : \overset{\text{tea}}{20\text{ lbs.}} :: \overset{\text{sugar}}{18\text{ lbs.}} : \overset{\text{sugar}}{x}$$

$$x = \frac{\overset{4}{\cancel{20}} \times \overset{6}{\cancel{18}} \times 8}{\cancel{3} \times \cancel{5}} = 192\text{ lbs.}$$

By the Chain Rule the same result may be obtained as follows:—

3 lbs. tea = 8 lbs. coffee.
5 lbs. coffee = 18 lbs. sugar.
x lbs. sugar = 20 lbs. tea.

$$x = \frac{\overset{6}{\cancel{8}} \times 18 \times \overset{4}{\cancel{20}}}{\cancel{3} \times \cancel{5}} = 192\text{ lbs.}$$

The several quantities are equated in pairs, *the first quantity in each pair being of the same kind as the second in the preceding pair*, thus forming a kind of chain, of which each pair forms a link.

Example II.—How many lbs. of coffee at 18d. per lb. should be given for 1 cwt. 1 qr. 24 lbs. of tea at 3s. 6d. per lb.?

$$\text{By Chain Rule}\ \left\{\begin{array}{l}\text{1 lb. coffee} = \text{18d.}\\ \text{42d. ... ...} = \text{1 lb. tea.}\\ \text{164 lbs. tea} = x\ \text{lbs. coffee.}\end{array}\right.$$

$$\text{By Proportion}\ \overset{\text{d.}}{18} : \overset{\text{d.}}{42} :: \overset{\text{lbs.}}{164} : \overset{\text{lbs.}}{x}$$

The calculation in either case will be the same.

$$x = \frac{\overset{7}{\cancel{42}} \times 164}{\underset{3}{\cancel{18}}} = \frac{1148}{3} = 382\tfrac{2}{3}\ \text{lbs.}$$

Example III.—How much Sterling (*i.e.* English money) will be required to pay an account of 4455 francs; the course of exchange being 24 frs. 75 c. per £1?

$$\overset{\text{francs}}{24.75} : \overset{\text{francs}}{4455} :: \overset{\pounds}{1} : \overset{\pounds}{x}$$

$$x = \frac{4455}{24.75} = \pounds 180$$

Example IV.—If the rate of exchange between London and Petersburg is 3s. 1½d. per rouble; between London and Amsterdam, 12 florins per £1; and between Amsterdam and Petersburg, 19 florins for 10 roubles. How much Sterling will be required to pay a debt of 1050 roubles (i.) remitted *direct:* (ii.) *indirect* by way of Amsterdam?

(i.) *Direct.*—$\overset{\text{d.}}{37\tfrac{1}{2}} \times 1050 = \overset{\text{d.}}{39375} = \overset{\pounds}{164}\ \overset{\text{s.}}{1}\ \overset{\text{d.}}{3}$

(ii.) *Indirect.*—

$$\left.\begin{array}{l}\pounds 1\ \ \ldots = 12\ \text{flor.}\\ 19\ \text{flor.} = 10\ \text{roub}\cdot\\ 1050\ \text{ro.} = \pounds x\end{array}\right\}\ x = \frac{19 \times 105\cancel{0}}{12 \times 1\cancel{0}} = \frac{1995}{12} = \pounds 166\ \text{5s. 0d.}$$

TABLE OF FOREIGN MONIES USED IN EXCHANGE.

France, Belgium, Switzerland.

1 Franc = 100 centimes = 9½d. nearly.

Par of Exchange = 25 francs 22½ cents. per £1.

Austria.

1 Florin = 60 kreuzers = 2s. 0½d. nearly.

Par of Exchange = 9 fl. 50 kr. per £1.

PRUSSIA.
1 Thaler = 30 groschen = 2s. 10¾d. nearly.
Par of Exchange = 6 th. 27 gr. per £1.
RUSSIA.
1 Rouble = 100 copecs = 3s. 1½d.
Par of Exchange = 6 roubles 40 cop. per £1.
ITALY.
1 Lira = 100 centesimi = 9½d. nearly.
Par of Exchange = 25 lire 22 c. per £1.
SPAIN.
1 Dollar = 20 reals = 4s. 2d.
Par of Exchange = 4 dol. 16 reals per £1.
PORTUGAL.
1 Milrea = 1000 reas = 4s. 9½d. nearly.
Par of Exchange = 4.175 milreas per £1.
HOLLAND.
1 Florin or Guilder = 100 cents = 1s. 8d.
Par of Exchange = 12 florins per £1.
DENMARK.
1 Rigsbank Dollar = 96 skillings = 2s. 2½d. nearly.
Par of Exchange = 9 R. d. 10 sk. per £1.
HAMBURG.
1 Mark banco = 16 schillings = 1s. 5½d. nearly.
Par of Exchange = 13 mk. 10½ sch. banco per £1.
TURKEY.
1 Piastre = 40 paras = 2⅙d. nearly.
Par of Exchange = 110 piastres per £1.
GREECE.
1 Drachma = 100 cents or lepta = 8¾d. nearly.
Par of Exchange = 28.15 dr. = £1.
UNITED STATES.
1 Dollar = 100 cents = 4s. 2d.
Par of Exchange = $4.87 per £1.
BRITISH INDIA.
1 Rupee = 16 annas = 1s. 10¼d. nearly.
Par of Exchange = 10⅓ rupees per £1.

N.B.—In mercantile language, the existence of a *par of exchange* is usually *assumed*. Such a par *may* exist between two countries making use of the same metal for their standard currency, but there can be no *invariable par* between two countries, one of which uses gold, and the other silver, as the standard; in such cases the *average course of exchange* over a long period may be taken as the *par of exchange.*

Ex. 84.

(1.) How many lbs. of coffee at 1s. 8d. per lb. should be given in exchange for 10 lbs. of tea at 4s. per lb. ?

(2.) How much paper at £1 12s. 6d. per ream must be bartered for 364 yards of cloth at 7s. 6d. per yard ?

(3.) How much linen at 3s. 4d. per yard should be given in exchange for 20 lb. of tea at 4s. 3d., and 7 dozen of sherry at 42s. ?

(4.) How many cwt. of rice at 44s. per cwt. must be given for 121 yards of cloth at 4s. 2d. per yard and £7 15s. 10d. in cash ?

(5.) Exchanged 2 cwt. 0 qrs. 19 lbs. of tea at 4s. 4d. per lb. and 8 dozen loaves of refined sugar each $19\frac{3}{4}$ lbs. for 165 lbs. of flour at 15s. per cwt. What was the price of the sugar per lb. ?

(6.) If 42 cwt. of cheese are bartered for 20 cwt. of hops at £4 18s. per cwt., what was the price of the cheese per lb. ?

(7.) If 9 English miles equal 8 Scotch ones, and 11 Irish miles equal 14 English ones, how many Scotch miles equal 99 Irish ones ?

(8.) If £1 sterling = 420d. Flemish; 58d. Flemish = 1 crown of Venice; 10 crowns Venice = 6 Venetian ducats; 1 ducat = 360 maravadies Spanish, and 272 maravadies = 1 Spanish piastre; how many piastres = £1000 sterling ?

(9.) If 3 lbs. of pepper be worth 4 lbs. of mustard, and 5 lbs. of mustard be worth 12 lbs. of candles, how many pounds of candles should be exchanged for 20 lbs. of pepper ?

(10.) If 7 lbs of coffee be worth 3 lbs. of tea, and 13 lbs. of coffee worth 48 lbs. of sugar, and 15 lbs. of sugar worth 28 lbs. of soap; how many lbs. of soap should be exchanged for 6 lbs. of tea ?

(11.) If 14 yards of cloth be worth 20 lbs. of sugar, and $11\frac{1}{4}$ lbs. of sugar be worth $13\frac{1}{4}$ lbs. of tea, and tea cost 3s. 6d. per lb.; what is the value of $7\frac{1}{2}$ dozen yards of cloth ?

(12.) If $\frac{1}{5}$ of a sheep be worth £$\frac{2}{3}$, and $\frac{3}{7}$ of a sheep be worth $\frac{1}{14}$ of an ox; how much must be given for 100 oxen ?

(13.) The mètre equals 39.37079 English inches, a Roman mile contained 1000 paces each 5 Roman feet, 1 Roman foot was .9708 of an English one; how many Roman miles are equal to 1 kilomètre ?

(14.) If 10 florins = £1 ; 25 thalers = 93 francs ; and 25 florins = 62 francs ; how many thalers equal £3 ?

(15.) If £5 = 126.50 francs ; 20 francs = 19.50 lire ; 6⅓ thalers = 11⅓ florins ; 25.22 lire = 6 thalers 20 gr. ; how many florins = £1 ?

Find the value of :—

(16). 7,497 piastres ; exchange at *par.*

(17.) 6706 rupees, 14 annas ; exchange at *par.*

(18.) $11.894 ; exchange $4.80 per £1.

(19.) 10,729 francs 50 cents ; exchange 24 fr. 97½ c. per £1.

(20.) 3015 florins 25 cents ; exchange 12¼ florins per £1.

What will be the *cost* in English money of the following :—

(21.) 15 hhds. of claret each 24.75 decalitres at 140.25 francs per hectolitre ? Exchange 25.10 frcs. per £1.

(22.) 50 boxes of French plums, each 5.75 kilogrammes at 1.80 fr. per kilog. ? Exchange 24.95 per £1.

(23.) 10,000 guilders Dutch 2½ per cent. stock, at 61½ per cent. ? Exchange at *par.*

(24.) 5000 thalers Prussian 4 per cent. stock, at 90⅜ per cent. ? Exchange 6⅔ thalers per £1.

(25.) 15,000 francs 4 per cent. rentes, at 105.80 francs per cent. ? Exchange 25.25 frcs. per £1.

(26.) 90 barrels of Archangel tar at 107 roub. 40 cop. per barrel ? Exchange 6 roub. 25 cop. per £1.

(27.) 27½ mètres of linen at 3.50 frs. per mètre ; 6¼ mètres, velvet at 17 frcs. ; 29 mètres, silk at 16.75 frcs. ; 28.75 mètres, merino at 4 frcs. ; 18 pairs of stockings at 3.50 frcs. per pair ; 7 pairs of gloves at 42 frcs. per pair ? Exchange 25.20 per £1.

(28.) 231 lbs. Habana cigars at $4.35 per lb. ; 63 lbs. Cubas at $6.25 ; 55 lbs. cheroots at $4.50 ; 62 lbs. old Silvas at $5.30 ? Exchange $4.90 per £1.

(29.) 647 galls. port wine at 2,750 reas per gallon ; 338 galls. ditto, at 3,200 reas per gallon ? Exchange 4s. 10½d. per milrea.

(30.) 9 cwt. 3 qrs. 14 lbs. cochineal at 3 rupees 11¾ annas per lb. ? Exchange 1s. 10¼d. per rupee.

XXXIV.—INVOLUTION.

INVOLUTION is simply the continued multiplication of a number by itself.

The product so obtained is called the **power** of the number or factor.

The number or factor multiplied is called the **root.**

The number of times the factor or number is repeated is called the **index** or **exponent** of the power.

Thus, 6^3 denotes $6 \times 6 \times 6 = 216$; here 6 is the *root*, 3 the *exponent* or *index*, and 216 the *power*.

The *second* power of a number is termed the **square**; thus the square of 7 or 7^2 is $7 \times 7 = 49$.

The *third* power of a number is called the **cube**; thus the cube of 9 or 9^3 is $9 \times 9 \times 9 = 729$.

Roots	1	2	3	4	5	6	7	8	9	10
Squares	1	4	9	16	25	36	49	64	81	100
Cubes	1	8	27	64	125	216	343	512	729	1000

When a number is to be involved to a high power, instead of multiplying the number by itself successively, we may multiply together those powers of the number the *sum of whose indices* is equal to the index of the required power.

Thus, to find the 12th power of 6, —

$6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 2176{,}782336.$

Or, $6^3 = 6 \times 6 \times 6 = 216.$

$6^6 = 6^3 \times 6^3 = 216 \times 216 = 46656.$

$6^{12} = 6^6 \times 6^6 = 46656 \times 46656 = 2176{,}782336.$

The square, cube, &c., of a *fraction* is found by raising both numerator and denominator to the required power.

Mixed numbers should be expressed as improper fractions.

Ex. 85.

Find the squares of:—

(1.) 27. (2.) 45. (3.) 365. (4.) 2345.
(5.) $3\frac{1}{7}$. (6.) $5\frac{1}{2}$. (7.) $9\frac{1}{2}$. (8.) $17\frac{5}{8}$.
(9.) 41.6. (10.) 54.32. (11.) 4.207. (12.) 8.3575.

Find the cubes of :—

(13.) 15. (14.) 31. (15.) 48. (16.) 67.
(17.) $7\frac{3}{5}$. (18.) $28\frac{1}{2}$. (19.) $17\frac{3}{4}$. (20.) $38\frac{3}{8}$.
(21.) 3.1. (22.) .046. (23.) 1.34. (24.) .0012.

XXXV.—EVOLUTION.

EVOLUTION, the reverse of Involution, is the process of finding the number which being *squared*, *cubed*, or *raised to any given power* will produce a given number.

The number thus found is called the **square root**, **cube root**, fourth root, &c., of the given number.

The most useful cases of Evolution are the **Extraction of the Square and Cube Roots.**

The symbol $\sqrt{\ }$ is used for square root, $\sqrt[3]{\ }$ for cube root, $\sqrt[4]{\ }$ for fourth root, and so on : thus, $\sqrt{4} = 2$; $\sqrt[3]{729} = 9$.

Roots are also often denoted by fractional indices, as $\frac{1}{2}$, $\frac{1}{3}$, &c., thus : $(36)^{\frac{1}{2}}$ or $\sqrt{36}$ means the square root of $36 = 6$.

Some of the higher roots of numbers *may* be obtained by successive extraction of lower roots, the *product of the indices* of which is equal to the index of the root to be extracted.

For instance, the fourth root may be found by taking the square root of the square root (for $4 = 2 \times 2$), the sixth root by taking the cube root of the square root, or *vice versâ* (for $6 = 3 \times 2$), &c., thus:— $\sqrt[4]{16} = \sqrt{4} = 2$; $\sqrt[6]{64} = \sqrt[3]{8} = 2$.

The same result may be thus written, using fractional indices :—

$$\sqrt[4]{16} = 16^{\frac{1}{2} \times \frac{1}{2}} = 4^{\frac{1}{2}} = 2\,; \quad \sqrt[6]{64} = 64^{\frac{1}{2} \times \frac{1}{3}} = 8^{\frac{1}{3}} = 2.$$

Numbers which have no exact roots are called **Surds.**

Only an *approximate* root of such numbers can be found, and this should be expressed in decimals.

1.—SQUARE ROOT.

RULE.—Divide the given number into periods of two figures beginning at the units' place.

Find the greatest square number contained in the first, or *left-hand*, period, and place its root in the quotient.

Subtract the square so found from the first period, and to the remainder annex the second period, for a dividend.

Double the root already found and place it as a divisor to the left of the dividend, find how many times it is contained in it, and annex the quotient figure both to the root and divisor.

Multiply the divisor so formed by the quotient figure just found, and subtract the product from the remainder.

Proceed thus till all the periods have been brought down, and if there be still a remainder, annex two ciphers and proceed as before, remembering to point off decimals as soon as the first period of decimals is brought down.

A full explanation of the principles on which the rule is based cannot be given without the aid of Algebra; the reason for pointing off the given number into periods of two figures may be thus explained:—

The square of any number under 10 consists of *not more than two* digits, thus $1^2 = 1$; $9^2 = 81$; between 10 and 100 of *not more than four* digits; thus $10^2 = 100$; $99^2 = 9801$; between 100 and 1000 of *not more than six* digits, and so on.

Hence the value of the first, or left-hand, figure of a root may be easily determined, for if the given number consist of *five* or *six* figures, we see the root must consist of *three*. Therefore, before commencing the calculation we point off the given number into periods of two figures *beginning at the units' place*, each of which periods corresponds to a figure in the root.

EXAMPLE.—Find the square root of 622521.

$$\begin{array}{r|l}
 & \dot{6}2\dot{2}5\dot{2}1 \;(789 \\
 & \underline{49} \\
148\,) & 1325 \\
 & \underline{1184} \\
1569\,) & 14121 \\
 & \underline{14121} \\
 & \cdots\cdots
\end{array}$$

Worked thus.—First, point off into 3 periods of 2 figures.

Then, since 49 is the greatest square number less than the first period 62, set down 49 under the 62.

Put 7 (the root of 49), in the quotient, subtract the 49 from 62, and to the remainder annex the next period, 25, for a new dividend.

Double 7, the root just found, and set the 14 so obtained, as an incomplete divisor to the left of the dividend, using it to find the next quotient, or root, figure 8; annex this 8 to both the 14 and 7.

Multiply 148 by 7 and subtract the product from 1325, and to the remainder annex the third period.

Double 78, the root already found, and use the 156 as an incomplete divisor to find the next root figure 9; annex this 9 to the 156 and 78.

Multiply the 1569 by 9 and subtract the product from 14121; as there is no remainder the original number is an exact square, the root of which is 789.

The working, as shown above, is all that is needful, the full process having been shortened by omitting ciphers and unnecessary figures, and bringing down period by period, similar to the process of Long Division.

If all the figures were written down with their local values, the working would stand thus:—

$$\begin{array}{rl}
 & 6\dot{2}2\dot{5}2\dot{1}\ (\ 700 + 80 + 9 \\
700 \times 700 = & \underline{490000} \\
(1400 + 80) = 1480\ ) & \overline{132521} \\
1480 \times 80 = & 118400 \\
(1400 + 160 + 9) = 1569\ ) & \overline{14121} \\
1569 \times 9 = & \underline{14121} \\
 & \cdots\cdots
\end{array}$$

If the number consist partly or entirely of a recurring decimal the recurring figure or figures must be brought down in successive periods of two figures instead of ciphers.

Ex. 86.

Find the square root of:—

(1.) 2304. (2.) 9216. (3.) 15876.
(4.) 19321. (5.) 46. (6.) 143.
(7.) .64. (8.) 1730.56. (9.) 2950.6024.
(10.) $17\frac{3}{8}$. (11.) $9\frac{43}{49}$. (12.) 5.875.
(13.) 6.6399. (14.) .064. (15.) $29\frac{52}{81}$.
(16.) $104\frac{1}{25}$. (17.) .000729. (18.) $7.6\dot{5}3\dot{1}$.

(19.) Find the side of a square which contains an area of 132496 mètres?

(20.) A square court contains 390.0625 mètres; find the length of one side.

2.—CUBE ROOT.

RULE.—Point off the given number into periods of *three* figures beginning at the units' place.

Find the greatest cube contained in the first or left hand period, subtract it from that period, and set its root in the quotient as the first figure of the required root.

Annex the second period to the remainder for a dividend.

For a divisor multiply the square of the root-figure already found by 300, and divide the dividend by it to obtain the next root-figure.

Cube the two figures of the root and subtract the result from the first two periods; and to the remainder annex the third period for a new dividend.

Divide this dividend by 300 times the square of the two root-figures already found, the quotient will be the third figure of the root.

Subtract the cube of the three figures in the root from the first three periods, bring down the next period for a dividend, and so on till all the periods are brought down, if there be still a remainder, periods of three ciphers may be annexed, and the root continued to any extent.

The divisor at each period will be 300 times the square of the root-figures already found.

Various processes for the extraction of the Cube Root have been given by different writers, the principles on which they are based cannot be fully understood without a knowledge of Algebra.

The reason for pointing off periods of three figures may be explained as follows :—

The cube of a number expressed by one *digit* consists of *not more than three* digits; the cube of a number expressed by *two* digits consists of *not more than six* digits, and so on; thus $1^3 = 1$, $9^3 = 729$; $10^3 = 1000$; $99^3 = 970299$. Hence each period of three figures in the number corresponds to one figure in the root.

Example.—Find the cube root of 2460375.

$$\begin{array}{rl}
 & \dot{2}46\dot{0}37\dot{5}\,(135 \\
 & 1 \\
\text{Divisor } 1^2 \times 300 = 300) & \overline{1460}\ \text{dividend} \\
 & \overline{2460} \\
13^3 = & \underline{2197} \\
\text{Divisor } 13^2 \times 300 = 50700) & \underline{263375}\ \text{dividend} \\
 & 2460375 \\
135^3 = & \underline{2460375} \\
 & \ldots\ldots\ldots
\end{array}$$

Cube roots being more easily found by the following process, than by the one just described, no examples have been given here; if needed they can be taken from page 181.

3.—Extraction of any Root.

This method, known as Horner's Method, is applicable to the extraction of any root.

It is, in fact, a mode of finding the numerical solution of any algebraic equation; the equation for the square root being of the form, $x^2 + 0x = a$; for the cube root, $x^3 + 0x^2 + 0x = a$; &c.

The examples already given are worked again by this method.

Example I.—Find the square root of 622521.

$$\begin{array}{llr}
1 & \underline{0} & 6\dot{2}2\dot{5}2\dot{1}\,(789 \\
 & 7 & \underline{49} \\
 & \underline{140} & 1325 \\
 & 148 & \underline{1184} \\
 & \underline{1560} & 14121 \\
 & 1569 & \underline{14121} \\
 & & \ldots\ldots
\end{array}$$

Example II.—Find the cube root of 2460375.

$$\begin{array}{llll}
1 & \underline{0} & \underline{0} & \dot{2}46\dot{0}37\dot{5}\,(135 \\
 & 1 & 1 & \underline{1} \\
 & 2 & \underline{300} & 1460 \\
 & \underline{30} & 399 & \underline{1197} \\
 & 33 & \underline{50700} & 263375 \\
 & 36 & 52675 & \underline{263375} \\
 & \underline{390} & & \ldots\ldots \\
 & 395 & &
\end{array}$$

Worked thus:—Arrange the work in as many columns as are indicated by *the exponent of the root plus one*, and place 1 at the head of the first column, the given number at the head of the last, and 0 at the head of the others.

These are the co-efficients of the unknown quantities in the algebraic equation; thus for *square root* the heads of the columns will be 1, 0, and the number; for cube root, 1, 0, 0 and the number; &c.

Point the given number into periods as already explained.

Step 1. The square-root of the first period forms the first root-figure.

Multiply the 1 in the first column by the root-figure (7), and *add* the product (7) to the 0 in the second column, multiply the 7 by the same root-figure 8 and *subtract* it from the first period, 62.

Returning to the first column again, multiply the 1 by the root-figure (7) and *add* it to the second column, which will make 14. This completes the 1st step.

Step 2. Multiply the number (14) in the second column by 10, bring down the next period in the third column, which will now be 1325.

Use the 140 as a trial divisor, which gives 8 as the second root-figure.

Then returning to the first column, 8 times 1 *added* to 140 = 148; 8 times 148 = 1184, subtracting 1184 from 1325 = 141.

Once more returning to the first column, 8 times 1 *added* to 148 = 156; this completes the second step.

Step 3. Multiply the number 156 in the second column by 10, and to the remainder 141 in the third column annex the third period making 14121.

Use the 1560 as a trial divisor which gives 9 as the *third* root-figure.

Then once more, 9 × 1 *added* to 1560 = 1569; and 1569 × 9 = 14121, this *subtracted* from the third column leaves nothing.

The required root is 789.

The operation for extracting the cube-root of a number is merely an extension of this process, all the steps being uniformly the same; thus in the second example :—

Step 1. The root of the first period 2 is 1; ∴ 1 is the first root-figure.

Once 1 *added* to 0 = 1; once 1 *added* to 0 = 1; once 1 *taken* from 2 = 1.

Once 1 *added* to 1 = 2; once 2 *added* to 1 = 3.

Once 1 *added* to 2 = 3.

Multiply the first column by 10; the second by 100; the third by 1000, or, as in this case, bring down the next period of three figures.

Step 2. Divide 1460 by the trial divisor 300 (remembering it will have to be increased to become the true divisor), it will give 3 as the second root-figure; then,

Three times 1 *added* to 30 = 33; *three* times 33 *added* to 300 = 399; *three* times 399 *taken* from 1460 leaves 263.

Three times 1 added to 33 = 36; *three* times 36 added to 399 = 507.

Three times 1 added to 36 = 39.

Multiply the first column by 10; the second by 100; the third by 1000, or, as in this case, bring down the third period.

Step 3. Divide 263375 by the trial divisor 50700, it will give 5 as the third root-figure; then,

Five times 1 added to 390 = 395; *five* times 395 *added* to 50700 = 52675; *five* times 52675 subtracted from 52675   0.

Therefore 789 is the required root.

From these examples it will be seen that the 1 stands *alone* in the first column:—that there are *two* columns of working in the square root, *three* in the cube root; and so on:—that for each step the first working column contains 2, 3, 4, &c., lines formed by addition, according to the index of the power; the second one line less, and so on; the last column containing only *one* line, which is to be *subtracted.*

The **thicker type** points to corresponding parts in the two processes.

EXAMPLE III.—Find the fifth root of 1470.08443.

1	0	0	0	0	$147\dot{0}.0844\dot{3}$ (4.3
	4	16	64	256	1024
	8	48	256	12800000	44608443
	12	96	640000	14869481	44608443
	16	16000	689827		....
	200	16609			
	203				

Ex. 87.

Find the cube root of:—

(1.) 2197. (2.) 9261. (3.) 19683. (4.) 531441.
(5.) 1815848. (6.) 2924207. (7.) 12812904. (8.) 3852.
(9.) 2. (10.) $2\frac{10}{27}$. (11.) $5\frac{104}{125}$. (12.) $7\frac{3}{8}$.

Find the following roots :—

(13.) 4th root of $\frac{16}{81}$. (14.) 4th root of 228886641.

(15.) 4th root of 97.41. (16.) 4th root of 12.

(17.) 6th root of 17161. (18.) 5th root of 708251.

(19.) 5th root of $\frac{32}{243}$. (20.) 5th root of $\frac{243}{16807}$.

(21.) 6th root of 1.159693. (22.) 7th root of 1.3608618.

(23.) The content of a cube is 13824 cubic centimètres, what is the length of one side?

(24.) A gallon contains 277.274 cubic inches, find the side of a cube which will contain exactly 8 gallons.

XXXVI.—DUODECIMALS, OR CROSS MULTIPLICATION.

In the common system of notation the local value of each figure increases in tenfold proportion in proceeding from right to left; hence it is called a **decimal system**, as shewn on page 2; the base or radix of the scale of numbers being 10, and the numeration ascending according to the powers of 10.

This, though an extremely convenient scale, is not the only possible one, for the local value of the digits might increase in *any proportion*. We might have counted by *fives* or *sixes*, for instance, instead of by *tens*, the scale would then have been a **quinary** or **senary** scale, and the base 5 or 6; while the local value of the digits would have increased from right to left according to the powers of those numbers.

In working addition, for instance, in the decimal scale, we carry 1 to the next column for every 10; in the quinary scale we should carry 1 for every 5; in the senary scale 1 for every 6; in the duodecimal scale 1 for every 12.

The subdivisions of an hour or a degree are on a sexagesimal (60) scale, descending by sixties;—thus, 1 hour or degree = 60 minutes; 1 minute = 60 seconds; 1 second = 60 thirds.

In our money tables we have instances of three different scales, thus we carry 1 for every 4 in the farthings, 1 for every 12 in the pence, and 1 for every 20 in the shillings.

Questions on the various scales of Notation, however interesting, are of little practical utility.

The most useful scale, after the common *denary* or **Decimal** one, is that which has 12 for its base, and is called the **Duodecimal** scale.

It is often employed in working questions of Mensuration, at least for the process of Multiplication, though all such questions may be very readily solved by vulgar fractions.

The process will be best shown by an example.

Example.—Multiply 44 ft. 10 in. by 16 ft. 11 in. (i.) by Duodecimals (ii.) by Vulgar fractions.

$$\text{(i.)}\quad \begin{array}{rrr} \text{ft.} & \text{in.} & \\ 44 & 10 & \\ 16 & 11 & \\ \hline 717 & 4 & \\ 41 & 1 & 2 \\ \hline 758 & 5 & 2 \end{array}$$

$$\text{(ii)}\quad 44\tfrac{10}{12} \times 16\tfrac{11}{12} = \frac{538}{12} \times \frac{203}{12} = \frac{109214}{144} = \overset{\text{sq. ft.}}{758}\ \overset{\text{sq. in.}}{62}$$

(i.) *Worked thus:*—16 times 10 = 160; 160 ÷ 12 = 13 and 4 remaining; set down 4 and carry 13.

(16 times 44) + 13 = 717.

11 times 10 = 110; 110 ÷ 12 = 9 and 2 remaining; set down 2 one place to the left, and carry 9.

(11 times 44) + 9 = 493; 493 ÷ 12 = 41 and 1 remaining; set down 1 and carry 41.

Then add the numbers together, the result = 758 *sq. feet,* 5 *parts* 2 *sq. in.* (or as each *part* = 12 *sq. in.*) 758 *sq. ft.* 62 *sq. in.*

Note here, that the expressions "feet multiplied by feet give square feet," and the like are *not strictly correct,* for in all multiplications the *multiplier must be an abstract number* denoting a number of repetitions; the meaning of such phrases really is, that the measures are to be multiplied together *as if* they were *abstract* numbers; and the product considered *as if* it were a *concrete* quantity, square feet, square inches, &c.

Ex. 88.

Find the surface measures from the following dimensions :—

(1.) 8 ft. 6 in. by 6 ft. 4 in.
(2.) 16 ft. 3 in. by 7 ft. 9 in.
(3.) 32 ft. 9 in. by 8 ft. 3 in.
(4.) 97 ft. 9 in. by 16 ft. 6 in.
(5.) 59 ft. 6 in. by 7 ft. 10 in.
(6.) 174 ft. 10 in. by 71 ft. 4 in.
(7.) 24 ft. 3 in. by 6 ft. 8 in.
(8.) 196 ft. 6 in. by 16 ft. 6 in.
(9.) 70 ft. 9 in. by 13 ft. 8 in.
(10.) 266 ft. 7 in. by 146 ft. 6 in.

XXXVII.—MENSURATION.

MENSURATION is the finding the **superficial extent** of areas of different forms, or the **cubic contents** of solid bodies.

A few only of the more common forms of surfaces and solids are given in the following examples, together with some applications of Involution and Evolution to the measurement of heights and distances ; the spaces traversed by falling bodies, &c.

1. SURFACES.

A SURFACE, **superficies**, or **area**, is measured by the number of times it contains a certain fixed area, as a square yard, square foot, &c.

A square yard, square foot, &c., means a surface equal to the area of a square, each side of which is respectively a linear yard, foot, inch, &c.

(i.) **Square** ... Area ... ... = Square of the length of one side.
" = Half the square of the diagonal.
(ii.) **Parallelogram**, Area ... ... = Length × perpendicular height or breadth.
(iii). **Triangle** ... Area ... ... $= \dfrac{\text{Base} \times \text{perpendicular height}}{2}$
(iv.) **Circle** ... ... Area ... ... = Diameter squared × .7854.
" = Circumference squared × .07958.
Circumference = Diameter × 3.14159.

N.B.—The areas of Circles are in the ratio of the *squares* of their *diameters*, and conversely, the diameters of circles are in the ratio of *the square roots* of their areas.

(v.) **Ellipse,** Area ... ... = Product of the two axes × .7854.
Circumference = Sum of the two semi-axes × 3.14159.

N.B.—The areas of Ellipses are in the ratio of the products of their respective axes or semi-axes.

(vi.) **Sphere,** Surface ... = (diameter)^2 × 3.14159.

(vii.) **Cylinder,** $\left\{\begin{matrix}\text{Convex}\\ \text{Surface}\end{matrix}\right\}$ = Diameter × height × 3.14159.

(viii.) **Cone,** $\left\{\begin{matrix}\text{Convex}\\ \text{Surface}\end{matrix}\right\} = \dfrac{\text{Diameter of base} \times \text{slant height} \times 3.14159.}{2}$

The *whole surface* of a **Cylinder** = the convex surface + the areas of the two ends.

The *whole surface* of a **Cone** = the convex surface + the area of the base.

Ex. 89.

(1.) How many square yards are there in a piece of ground 864 ft. 3 in. long, and 62 ft. 6 in. wide?

(2.) The side of a court-yard measures 168 feet; what will it cost paving at 2s. 6d. per square yard?

(3.) The Great Pyramid stands on a base 693 feet square; (i.) how many *acres*, (ii.) how many *hectares* does it cover?

(4.) Find the area of a square whose side is 132 mètres, and express it in English measures.

(5.) Find the area of a triangle, of which the base measures 7136 links and the perpendicular 790 links.

(6.) How many square yards are there in 5 hectares; an *are* being a square decamètre?

(7.) Taking the circumference of the earth at 25000 miles, what is its diameter?

(8.) If the moon's diameter is 2180.418 miles, what is its circumference?

(9.) How many square yards are there in a circle whose diameter is 35 feet?

(10.) What is the area of a circle whose circumference is 12 ft.?

(11.) How many times is a circle 6 ft. in diameter as large as another 4 ft. in diameter?

(12.) Find (i.) the area, (ii.) the circumference of a circle whose diameter is 15 mètres.

(13.) Find (i.) the area, (ii.) the circumference of an ellipse whose axes are 30 ft. and 20 ft.

(14.) What is the difference (i.) in area, (ii.) in circumference between a circle whose diameter is 50 ft., and an ellipse whose axes are 60 ft. and 40 ft.?

(15.) The area of a circle is 113.0976 square yards, what will be the diameter of another the area of which is four times as large?

(16.) What was the expense of making a walk round a circular garden, at 2s. 6d. per square yard; the diameter of the garden being 525 ft., and the breadth of the walk 21 ft. 6 in.

(17.) The major axis of an elliptical grass-plot measures 25 yards, and the minor 15 yards; what was the cost of paving a walk round it, with Portland stone, at 3s. 6d. per square yard; the breadth of the walk being 5½ yards?

(18.) Find, both in French and English measures, (i.) the *convex surface*, (ii.) the *whole surface* of a cone, the slant height being 50 mètres, and the diameter of the base, 8.5 mètres.

(19.) Find the surface of a globe 1.5 mètres in diameter, in both French and English measures.

(20.) Find (i.) the *convex surface*, (ii.) the *whole surface* of a cylinder 10 ft. high and 18 inches in diameter.

2. SOLIDS.

A **Solid** is measured by the number of times it contains a certain fixed solid, as a cubic yard, cubic foot, &c.

A cubic yard, cubic foot, &c., means a solid equal to the content of a cube each side of which is respectively a lineal yard, foot, &c.

(i.) **Cube** ... . Contents = cube of the length of one side.

(ii.) **Prism** or **Cylinder**, Contents = area of end × perpendicular height.

(iii.) **Pyramid** or **Cone**, Contents area of base × ⅓ the perpendicular height.

(iv.) **Sphere** ... ... Contents = diameter cubed × .5236.
= circumference cubed × .01688.

N.B.—A Cone is ⅓ and a sphere ⅔ of the circumscribing Cylinder, *i.e.* of a Cylinder of the same height and diameter.

The contents of spheres are in the ratio of the *cubes* of their diameters; and conversely, the diameters of spheres are in the ratio of the *cube roots* of their solidity.

Ex. 90.

(1.) Find (i.) the number of cubic feet in a cistern, 3 ft. 2 in. long, 2 ft. 8 in. broad, and 2 ft. 6 in. deep; (ii.) the number of gallons of water it will hold; 1 gallon being equal to 277¼ cubic inches.

(2.) Find the solid content of a cylindrical column 20 ft. high, and 5½ feet in circumference.

(3.) Find the number of cubic yards in a square pyramid, each side of which is 90 ft. long, and the perpendicular height 75 ft.

(4.) How many 3 inch cubes can be cut out a 12 inch cube?

(5.) Find the solid contents of a cylinder, 50 ft. high, and 6 ft. in diameter.

(6.) What is the content of a cone 100 ft. high, the base of which is 12 ft. in diameter.

(7.) Find the solidity of a sphere whose diameter is 2.4 mètres.

(8.) What is the diameter of a globe 6 times as large as another 20 ft. in diameter?

(9.) Find (i.) in inches, (ii.) in centimètres, the side of a cube containing exactly 1 litre.

(10.) The solidity of a sphere is 7238.2464 cubic decimètres, what is its diameter?

3. HEIGHTS, DISTANCES, &C.

In a right-angled triangle, the square on the *hypothenuse* or side opposite the right angle, is equal to the sum of the squares on the other two sides, *i.e.* the *base* and *perpendicular*. Hence,

$$\text{Hypothenuse} = \sqrt{(\text{Base})^2 + (\text{Perpendicular})^2}$$

$$\text{Base} \ \ldots \ \ldots = \sqrt{(\text{Hypothenuse})^2 - (\text{Perpendicular})^2}$$

$$\text{Perpendicular} = \sqrt{(\text{Hypothenuse})^2 - (\text{Base})^2}$$

A *mean proportional between two numbers* is a third number which bears the same ratio to the smaller of the given numbers as the larger bears to such third number.

Thus 14 is a mean proportional between 4 and 49, for,

$$4 : 14 :: 14 : 49$$

$$(14)^2 = 4 \times 49 \therefore 14 = \sqrt{4 \times 49}.$$

Hence the mean proportional between two numbers is the square root of their product.

The **whole distance** or **space** through which a body falls from a state of rest in a given time, is equal to 16.1 *feet multiplied by the square of the number of seconds.*

The **number of seconds** in which a body falling from rest traverses a certain distance or space, $= \sqrt{\dfrac{space}{16.1}}$

The **velocity** acquired, at the end of any number of seconds, by a body falling from rest, is equal to 16.1 *feet multiplied by twice the number of seconds.*

These three relations may be expressed by the following equations:—

$$s = 16.1 \times t^2\,; \quad t = \sqrt{\frac{s}{16.1}}\,; \quad v. = 16.1 \times 2t.$$

Ex. 91.

Find a mean proportional between the following:—

(1.) 48 and 147. (2.) $5\frac{1}{16}$ and $5\frac{4}{9}$.

(3.) 12.5 and 12.25. (4.) $7\frac{3}{5}$ and 18.71.

(5.) The side of a square is 7 ft., what is the diagonal or distance across from corner to corner?

(6.) Find the diagonal of a floor 56 ft. long, 42 ft. broad?

(7.) Required the length of a scaling ladder to reach the top of a wall whose height is 32 ft.; the breadth of the moat before it being 44 ft.?

(8.) The distance from the ridge to the eaves of a building is 15 ft, and the height of the gable 9 ft.; what is the breadth of the building?

(9.) A line stretched from the top of a cliff to a point 147 feet from the base measured 525 ft., what was the height of the cliff?

(10.) What must be the length of a ladder, which, standing 10 ft. 6 inches from the upright of a building, will reach a window 18 ft. 6 inches from the ground?

(11.) A. travels due north 24 miles per day for 3 days, and then east 26 miles a day for 2 days, how far is he from the starting place?

(12.) Find the diagonal of a floor 9 mètres long and 6 mètres wide.

(13.) A cord 97 mètres long is stretched to the ground from the top of a tower, 50 mètres high, how far from the foot of the tower is the lower end of the cord?

(14.) Find the space traversed by a body falling during 3, 5, 7, seconds, and the velocity at the end of each period.

(15.) What time will a body take in falling 500, 800, and 1000 ft.?

(16.) With what velocity does a body reach the ground after falling (i.) for $2\frac{1}{2}$ seconds, (ii.) $4\frac{1}{4}$ seconds, (iii.) through 257.6 feet?

XXXVIII.—MISCELLANEOUS EXAMPLES.

Ex. 92.

(1.) What sum must be added to £31 14s. $6\frac{1}{2}$d. to make it exactly divisible by £2 11s. 4d.?

(2.) Reduce 53 tons 4 cwt. 77 lbs. to ounces; and 1 mile 5 furl. 13 poles 2 inches to inches.

(3.) What is the value of 687 Napoleons, a Napoleon being worth 15s. $11\frac{1}{2}$d.?

(4.) Find the value, at £3 5s. per cwt. net, of 100 bags of coffee, weighing gross 145 cwt. 1 qr.; tare 2 lbs. per bag, and draft 2 lbs. per bag.

(5.) Simplify:—

(i.) $\frac{7}{11} + \frac{15}{17} - \frac{16}{51} + \frac{18}{45} - \frac{19}{68} + \frac{23}{67}$.

(ii.) $3\frac{11}{108} + 7\frac{5}{36} + 4\frac{5}{12} - 12\frac{17}{54} - \frac{13}{27}$.

(6.) Simplify:—

(i.) $5\frac{1}{11}$ of $1\frac{1}{14}$ of $4\frac{8}{9}$ of 3.

(ii.) $1\frac{1}{4} \times \frac{5}{7}$ of $6\frac{1}{10}$ of $\frac{14}{13} \times 2\frac{4}{15} \times 1\frac{16}{17}$.

(7.) Find the value of $\frac{4}{5}$ of $\frac{6}{7}$ of £49 4s. $4\frac{1}{2}$d.

(8.) Divide .00456 by .1458, and .3456 by .0054; also (.0372 + .0459) by (.3456 — .0126).

(9.) Find the product of 270.26 × .00025; also of $(.64)^2 \times (.012)^3 \div .576 \times .1728$.

(10.) Reduce 3 qrs. 18 lbs. $10\frac{2}{3}$ ozs. to the fraction and decimal of a cwt., and 13s. $1\frac{1}{2}$d. to the fraction and decimal of £4 11s. $10\frac{1}{2}$d.

(11.) Find the sum of .35 tons, .39 cwt. and .29 qr.

(12.) Find the sum of £.121875 + 17s. 6.75d.

(13.) Find the avoirdupois weight of the following casks; No. 1, 84 kilog. 35 decagr.; No. 2, 93 kilog. 3 decagr.; No. 3, 75 kilog. 64 decagr.

(14.) Find the value of 24684 lbs. at £5 2s. 6½d. for each 50 lbs.

(15.) A. sells to B. $\frac{7}{8}$ of $\frac{1}{3}$ of $\frac{4}{5}$ of 30 sheep for $\frac{3}{14}$ of $\frac{9}{28}$ of $\frac{8}{9}$ of £210; what was the average price of each sheep?

(16.) A. and B. engage in trade, and gain £521 8s. 2d.; A. put in £250 for 6 months, B. £300 for 5 months, C. £400 for 12 months; required each partner's share of the gain.

(17.) If £6 13s. be the wages of 8 men for 3.25 days, what will be the wages of 20 men for 9.25 days?

(18.) If a family of 13 persons spend £64 in butcher's meat, in 8 months, when meat is 6d. per lb., how much money, at the same rate, should a family of 12 persons spend in 9 months, when meat is 6½d. per lb.?

(19.) Find the cost of insuring property worth £1240 16s. 8d. at £3 1s. 8d. per cent.

(20.) Find the simple interest on £379 17s. 2¼d. for 3 years 9 months, at 3 per cent.; also on £1374 6s. 4d. for 2 years 75 days, at £3 13s. per cent.

(21.) Find the compound interest on £699 17s. 4¾d. in 3 years, at 4 per cent.

(22.) The receipts of the Great Eastern Railway, for the week ending October 2nd, amounted to £16,286, against £15,991 in the corresponding week of last year. Find the increase per cent.

(23.) A person engages to pay a debt of £2330 as follows: £630 in 5 months, £300 in 6 months, £500 in 7 months, and the remaining £900 in 8 months; but he afterwards proposes to pay the whole in one sum; in what time should it be paid?

(24.) Find the present value of £93 1s. 3¼d. due 6 months hence, at 4 per cent.

(25.) If a person purchase £4000 stock at 93⅜, what loss will he sustain by selling out at 92⅞?

(26.) A grocer buys 200 lbs. of sugar at 5¼d. per lb.; what does it cost him? At what price per lb. must he sell it to clear £1 9s. 2d.?

(27.) If 161 lbs. of tea are given in exchange for 12 cwt. 21 lbs. of sugar at 5¾d. per lb., what is the price of tea per lb.?

(28.) Extract the square root of 978121; .00081; and $\frac{5.04}{.021}$.

(29.) Multiply together 21 feet 5 inches, 3 feet 3 inches, 2 feet, 7 inches, by duodecimals.

(30.) Find by duodecimals the capacity of a cube whose edge is 2 ft. 8 in., and prove the truth of the result by vulgar fractions.

Ex. 93.

(1.) A person, after paying 10d. in the pound income tax upon a yearly income of £400, laid by £85 17s.; what was his average weekly expenditure?

(2.) Reduce 3 acres 20 rods 12 sq. yards 7 sq. feet to feet.

(3.) Find the value of:—

(i.) 1296 articles, at 16s. $10\frac{1}{2}$d. each;
(ii.) 4976 yards, at £2 11s. $8\frac{3}{4}$d. per yard.

(4.) What sum will be required to purchase a piece of land containing 75 acres 1 rood 30 perches, at £26 18s. 4d. per acre?

(5.) Required the net weight of 29 barrels, each weighing 2 cwt. 3 qrs. 21 lbs., allowing $5\frac{3}{4}$ lbs. per barrel for tare.

(6.) Simplify:—

(i.) $\frac{1}{8} + \frac{11}{16} + \frac{13}{28} - \frac{57}{112} + \frac{3}{11} - \frac{37}{56}$;
(ii.) $3\frac{16}{35} + 5\frac{5}{15} + 7\frac{13}{35} - 12\frac{3}{7} + 4\frac{10}{12}$;
(iii.) $4\frac{1}{2} - 3\frac{3}{8} + 2 + 8\frac{5}{6} - 4\frac{1}{2} + \frac{1}{6} + 1.75 + \frac{1}{4} + \frac{3}{4} - \frac{7}{8}$.

(7.) What is the least sum of money which can be paid exactly in either florins, crowns, or guineas?

(8.) Reduce 14s. $0\frac{3}{4}$d. to the fraction of £1 2s. 6d.; and find the value of $\frac{7}{8}$ of half-a-guinea — $2\frac{1}{12}$ of half-a-crown.

(9.) Divide (.3146 — .1422) by (.4149 + .342); also $\frac{1}{5}$ (.0414 + .00414) by $\frac{5}{7}$ (.0135 — .00135).

(10.) Divide the product of 21.35, .00721, .011 by the product of 1.1 and .00448.

(11.) Reduce £1 3s. $5\frac{1}{4}$d. to the decimal of 10s., and 3 qrs. 3 lbs. 1 oz. $12\frac{4}{5}$ drs. to decimal of a cwt.

(12.) In a manufactory 40 boys, 80 women, and 130 men are employed; the boys receive 1.75s. per day, the women twice, and the men three times as much as a boy; find the wages earned in a year.

(13.) Find the sum of .6 of 6s. 8d., $\dot{7}1428\dot{5}$ of £4 7s. 6d., and $.\dot{8}\dot{1}$ of £2 7s. $2\frac{1}{2}$d.

(14.) Divide 406 litres of soup among 36 men and 44 women, giving each man twice as much as each woman.

(15.) A train going 27 miles per hour, performs a certain distance in 3 hours 20 minutes; how long would a train going 36 miles per hour *take to perform half* the distance?

(16.) If three persons own a ship, and the share of the first is $\frac{3}{7}$ of the whole, that of the second $\frac{5}{10}$, and that of the third £535$\frac{5}{7}$, what is the worth of the ship?

(17.) If .6 of a yard cost £.58$\dot{3}$, what will .4 of an English ell cost?

(18.) If the wheel of a railway carriage be 12 feet in circumference, how many times will it revolve in a minute in going, in 6 hours 36 minutes, from London to Manchester, which is 210 miles?

(19.) Divide £32 1s. 8d. among 4 persons, in the proportion of the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$.

(20.) If 375 acres of land let for £401 11s. 3d., what is that per acre? and what will be the rent per acre if raised 12$\frac{1}{2}$ per cent.?

(21.) At what rate per cent. will the interest on £375 amount to £48 2s. 6d. in 3 years 8 months?

(22.) Find the amount of £1000 in 3$\frac{3}{4}$ years at 5 per cent. per annum compound interest.

(23.) The amount of money expended for the maintenance of the poor in England and Wales, during the year ending Sep. 29, 1850, was £3469857, and during the year ending Sep. 29, 1851, £3288192; what was the decrease per cent.?

(24.) I owe 3 bills, one due on June 29, amount £150; one due August 21, amount £100; and one due December 31, 1864, amount £200; this being the 20th May, what is the equated time of payment of the whole?

(25.) A bill for £500 was due Feb. 2, 1865; of this sum, £80 was paid, March 9; £115, May 15; £25, June 1; and the balance, Aug. 14; what interest was then due at 5 per cent.?

(26.) Which is the better investment, the 3 per cents. at 90, or the 3$\frac{1}{2}$ per cents. at 102? What income would be derived from the investment of £2499 15s. in the former?

(27.) If an article at the retail price of 4s. 1$\frac{1}{2}$d. per lb. lose 12 per cent., what must it be sold for to gain 4 per cent.?

(28.) A thaler being equivalent to 3s. 1$\frac{1}{2}$d., and £1 sterling to 25.6 francs; how many francs are worth 1000 thalers?

(29.) Find the square root of 1635841, and the cube root of 2 to 3 places of decimals.

(30.) A playground is 100 ft. 9 in. long, and 45 ft. 3 in. wide; find the area.

(31.) How many square feet of board will be required to make a cubical box, which shall contain 81.$\dot{3}7\dot{0}$ cubic feet?

Ex. 94.

(1.) How many bricks, each 9 inches long, $4\frac{1}{2}$ inches wide, 3 inches thick, will be required to build a wall 25 feet long, 18 feet high, 2 feet 6 inches thick?

(2.) Reduce 13 lbs. 11 oz. avoirdupois to troy weight.

(3.) Find the value of :—

(i.) 7 cwt. 3 qrs. 16 lbs. at £4 15s. 9d. per cwt.;
(ii.) 20 acres 3 rd. 25 per. at £5 7s. $6\frac{1}{2}$d. per acre;
(iii.) 135 sq.yards 6 ft. 72 in., at 2s. 3d. per sq. yard.

(4.) Find the cost of 25 bags pimento, weighing gross 22 cwt. 2 qrs. 15 lbs., draft 1 lb. per bag, tare 4 lbs. per cwt., at £3 15s. 6d. per cwt.

(5.) Simplify :—

(i.) $2\frac{1}{2} + 5\frac{1}{4} + 10\frac{5}{36} + 20\frac{1}{128}$;
(ii.) $3\frac{7}{48} - 11\frac{5}{8} + 12\frac{7}{24} - 1\frac{7}{36}$.

(6.) Multiply $3\frac{1}{3}$ by $18\frac{9}{20}$, and divide $\frac{7}{8}$ of 6 by $\frac{3}{4}$ of $\frac{6}{7}$ of $4\frac{1}{12}$.

(7.) Three men start walking round a field $2\frac{1}{2}$ miles in circumference, at the rates of $4\frac{1}{2}$, 5, and 3 miles an hour respectively; in what time will they all arrive together at the starting place.

(8.) Find the sum of .63 of $33\frac{1}{3}$ + $10\frac{2}{3}$ of .51875 and $3\frac{5}{16}$ of .006.

(9.) Multiply 130.8534 by 1.14, and divide the product by .0076.

(10.) Express the value of $\frac{3}{8}$ of a guinea + $\frac{3}{16}$ of a crown + $\frac{3}{10}$ of 7s. 6d. — $\frac{3}{8}$ of 2d. as a decimal of 16s.

(11.) Find the sum of .75 of 2.05s., £.634375, .025 of 25s., and $.31\dot{6}$ of 30s.

(12.) A farmer buys 50 lambs for 1830 frcs. 50 cents., and 60 more for 2841 frcs. 25 cents, how much will he gain or lose by selling them at 43 frcs. 5 cents each?

(13.) If 10 horses eat 450 kilog. of hay in 6 days, how many kilogrammes will be required for 17 horses for 11 days?

(14.) A person buys an annuity of £3 12s. for £106 19s. What annuity could he buy for £5013 5s. $7\frac{1}{2}$d.?

(15.) A ship and cargo are worth £10892 9s. 6d., the ship is worth $\frac{2}{3} + (\frac{4}{15}$ of $\frac{7}{16}) - \frac{7}{80}$ of the whole; what is the value of $\frac{4}{11}$th of the cargo?

(16.) If an ounce of gold be worth £$4.\dot{0}09\dot{9}$, what is the value of $.56\dot{1}$ of a lb.?

(17.) If 9 men build a wall in $30\frac{5}{8}$ days, of $6\frac{2}{3}$ hours each; how long will it take 12 men, at $12\frac{1}{3}$ hours per day, to do $3\frac{3}{4}$ times as much?

(18.) A person has but £100 to pay the following sums; namely, to A, £48 15s.; to B, £72 10s.; and to C, £84 13s. 4d.; what share ought to be paid to each?

(19.) A cargo, worth £3800, is to be insured at 5 per cent.; for what amount must the insurance be effected, so that, in case all should be lost, the owners may recover both the value and the premium paid?

(20.) Find the interest on £1235 15s. 6d. for $4\frac{1}{2}$ years, at $3\frac{3}{8}$ per cent.; also on £450 10s. at $5\frac{1}{4}$ per cent. for $3\frac{1}{2}$ years.

(21.) What principal will produce a yearly interest of £341 5s. at 5 per cent.?

(22.) Of 450,000 letters posted in London, 8 per cent. were unpaid, 211,140 were stamped, what percentage of the paid letters was prepaid in money?

(23.) A bill of £170 was due August 12, and left unpaid till Sept. 18, when £54 was paid; the balance remained till Oct. 17, when £56 was paid; and, finally, the whole was settled Nov. 14; how much interest was due at 5 per cent.?

(24.) Find the discount and present worth of £1062 10s. for 16 months at $3\frac{3}{4}$ per cent.

(25.) A person possesses £3200 in the 3 per cents., which he sells at $99\frac{3}{4}$; he invests the proceeds in railway shares at £56 a share, which shares pay 5 per cent. interest on £45, the amount paid on each share. How much is his income altered by the transaction?

(26.) Find (i.) the cost of 3 chests of tea, each weighing 2 cwt. 2 qrs., at 4s. $10\frac{1}{2}$d. *per pound*, allowing 28 lbs. tare to each chest; and (ii.) the selling price, so as to gain 20 per cent.

(27.) What is the price of a horse, when $4\frac{1}{2}$ geese are worth $1\frac{1}{17}$ of a sheep, and $4\frac{4}{17}$ sheep are worth $5\frac{1}{10}$ calves, and $6\frac{1}{4}$ calves are worth $1\frac{1}{5}$ oxen, and $2\frac{1}{4}$ oxen are worth $1\frac{8}{17}$ horses, and one goose be worth 10s.?

(28.) Extract the square root of 24383844; and the cube root of 2406104.

(29.) How much paper $\frac{5}{8}$ of a yard wide would be required for the walls of a room 20 feet square, and 12 feet high, having 3 windows each 8 feet by 3 feet 6 inches? What would be the cost of papering it at $4\frac{1}{2}$d. per yard.

(30.) A cubic mass of metal is worth £68279 8s. 8d., at £4 3s. 4d. per cubic inch. Find the area of one of its faces.

Ex. 95.

(1.) If 15,000 copies of the *Times* be issued daily, each copy consisting of 2 sheets, and each sheet being 4 ft. by 3 ft., how many acres will one edition cover?

(2.) A person buys 3 hect. 40 ares of land for 12,500 francs, at what price must he sell it per hectare to gain 1100 francs?

(3.) How much money will pay for 196 qrs. of wheat at £2 15s. 6d. per quarter, and 109½ qrs. of oats at £1 17s. 11d. per quarter?

(4.) Find the total cost of the following parcels of sugar:—

14, each	3 cwt. 1 qr. 15 lbs.,	tare 15 lbs.	per cwt.,	at £3 11s. 3d.	per cwt.
11 ,,	3 cwt. 3 qrs. 19 lbs.,	,, 16½ lbs.	,,	at £3 11s. 7d.	,,
19 ,,	3 cwt. 1 qr. 17 lbs.,	,, 15½ lbs.	,,	at £3 12s. 2d.	,,
7 ,,	3 cwt. 1 qr. 23 lbs.,	,, 16¼ lbs.	,,	at £3 12s. 5½d.	,,

(5.) If A. can mow 2 acres in 3 days; B. 4 acres in 5 days, and C. 13 acres in 12 days; how long will they be altogether in mowing 17 acres?

(6.) A. and B. together can reap a field in 12 days, B. and C. together in 13 days; but if a boy D. help them, they can finish the work as soon as A. and B. together; in what time could D. alone do it?

(7.) Find the G. C. M. of 14112 and 22176; and the L. C. M. of 108, 207, and 216.

(8.) What are the values of $\sqrt{27\frac{9}{16}} - \sqrt{9\frac{49}{64}}$, and $\sqrt{\frac{2704}{4225}} - \sqrt{\frac{2304}{5184}}$?

(9.) Simplify:—

$$\left(\frac{3\frac{1}{7}}{4\frac{7}{12}} \text{ of } \frac{7\frac{1}{5}}{3\frac{3}{11}} \text{ of } \frac{7\frac{1}{24}}{9\frac{1}{63}}\right) \div \left(\frac{17\frac{1}{2} \text{ of } 3\frac{3}{7}}{3\frac{1}{4}} - \frac{11\frac{1}{4} \text{ of } 3\frac{1}{5}}{4\frac{1}{5}}\right) \times \frac{13}{6\frac{5}{11}} \text{ of } \frac{117}{790}.$$

(10.) Reduce £1 3s. 7½d. to the decimal of £5, and $.1\dot{6}$ of 1 acre 1 ro. 25 po. to the decimal of 2 acres 1 ro. 15 po.

(11.) Add together .0625 of £2, .003 of £10⅛ and $.\dot{1}4285\dot{7}$ of £7 14s. 7d.

(12.) A bankrupt pays 5 florins 75 mils. in the pound; how much will a creditor lose on a debt of £11793 5 fl. 25 mils.?

(13.) A person buys 76 kilog. of candles at 1 fr. 30 c. per kilog., and some soap at 1 fr. 50 c. per kilog. The bill came to 173 frs. 80 c.; how much soap did he buy?

(14.) What will a piece of plate cost, which weighs 3 lbs. 4 oz. 5 dwts., if the price of silver be £3 6s. per lb., and the charge for workmanship be 2s. 8d. per ounce?

(15.) If 8 $\dot{3}\dot{6}$ lbs. cost £.1875, what is the value of $6.1\dot{6}$ lbs.?

(16.) If a room be 13.5 feet long and 12.5 feet wide; what will be the expense of carpeting it at 10.625s. per yard, the carpet being 30 inches wide?

(17.) If $4\frac{2}{3}$ fields of $18\frac{3}{4}$ acres each can be mowed by 7 machines in half a day; how many fields of $28\frac{1}{8}$ acres each might be mowed by 6 machines in 3 days?

(18.) The number of passengers by a certain railway train was 135; the first, second, and third class passengers being respectively in number as 4, 9, and 14. The average number of miles traversed by the various classes was 37, 24, and 19; and the fares 2d., $1\frac{1}{2}$d., and 1d. per mile. Find the whole sum earned by the train.

(19.) For what sum must an insurance be effected, in order to cover a loss of £3500, together with the premium of £3 8s. per cent.?

(20.) What sum will amount to £479 6s. 6d. in $2\frac{3}{4}$ years, at $3\frac{1}{4}$ per cent. simple interest?

(21.) Find the difference between the simple and compound interest on £550 for 4 years at 5 per cent.

(22.) Find the average of 3.065, 82, $34\frac{3}{10}$, $10\frac{1}{2}$, $21.\dot{6}$, $18\frac{7}{12}$, and $5.\dot{2}8571\dot{4}$.

(23.) £240 is to be paid as follows: £60 in 60 days, £80 in 96 days, £40 in 250 days, and the rest in a year and 35 days: required the equated time for paying the whole?

(24.) Find the difference between the interest and discount of £625 15s. for 8 months, at $3\frac{3}{4}$ per cent. per annum.

(25.) A person sells sufficient 3 per cent. consols at 96, to realize £5096, and invests this money in the India 5 per cent. loan at 104. Find the alteration in his income.

(26.) A person, by selling an article for 14s., clears $\frac{3}{8}$ of the money; what will he gain per cent. if he raises the price to 17s. 6d.?

(27.) If $3\frac{4}{5}$ sheep be worth $1\frac{1}{15}$ of a horse, and $11\frac{1}{5}$ horses be worth $17\frac{1}{17}$ bullocks, and $3\frac{5}{8}$ bullocks be worth $2\frac{1}{5}$ quarters of wheat, and $3\frac{1}{3}$ quarters of wheat be worth $4\frac{1}{4}$ quarters of barley, and barley be worth 4s. $10\frac{1}{2}$d. per bushel; what is the price of 560 sheep?

(28.) Find the square root of .023409, and the cube root of 3581577 and .005451776.

(29.) If 8 guineas be expended in purchasing Brussels carpet $\frac{3}{4}$ yd. wide, at 3s. 6d. per yard, for a room 20 ft. long and 16 ft. 9 in. broad; how much of the floor will remain uncovered?

(30.) A canal was to be made in 36 days, exactly 1 mile long, with a depth of 16 feet of water, with a width at the surface of 16 yards, and at the bottom of 8 yards, the sides sloping equally to the bottom; what would be the price of the excavation at $6\frac{1}{4}$d. per cubic yard? Also it having been found that at the end of 20 days only $\frac{3}{11}$ of the work were completed, although 300 men had been working 8 hours per day; what extra number of men, all working 10 hours per day, must be put on to finish the work in the stipulated time?

Ex. 96.

(1.) The bars of an iron railing are 5 feet 10 inches high. If laid down end to end, they would extend over a distance of 23 miles 600 yards; how many rails are there?

(2.) How many planks $15\frac{1}{4}$ feet long $1\frac{1}{4}$ feet wide will floor a room 60 feet long by $30\frac{1}{2}$ feet wide?

(3.) A person laid out a part of £116 9s. 2d. in the purchase of 684 yards of linen at 3s. $1\frac{1}{2}$d. per yard, and the rest in calico at $11\frac{1}{2}$d. a yard; how many yards of calico did he buy?

(4.) Find the price of 3 cwt. 2 qrs. 16 lbs. of sugar, at 67s. 8d. per cwt.; and the price per lb., when 4 cwt. of tea cost £68 12s.

(5.) How many gallons *nett* are there in 21 casks of oil, the *average gross weight* of each being $3\frac{1}{2}$ cwt.; tare 15 lbs. per cwt., and 1 gallon weighing $7\frac{1}{2}$ lbs.?

(6.) Simplify:—

$$\frac{3\frac{1}{7}}{4\frac{7}{12}} \text{ of } 6\frac{3}{13} \text{ of } \frac{7\frac{1}{24}}{9\frac{1}{63}} \div \left(\frac{17\frac{1}{2} \text{ of } 3\frac{3}{7}}{3\frac{1}{4}} - \frac{11\frac{1}{4} \text{ of } 3\frac{1}{2}}{4\frac{1}{3}}\right) \times \frac{39}{71} \text{ of } 1\frac{3}{140}.$$

(7.) Reduce $\frac{3}{5}$ of $(2\frac{3}{7} - \frac{8}{9})$ of a lb. troy, to the fraction of $\frac{2}{3}$ of $(\frac{11}{10} + \frac{4}{63})$ cwt. avoirdupois.

(8.) Find the value of $\frac{3\frac{7}{11}}{7\frac{1}{2}}$ of $\frac{10\frac{5}{7}}{540}$ of $\frac{77}{4\frac{2}{7}}$ of £$1\frac{7}{20}$.

(9.) Divide (i.) (.0045 + .045) by (.0432 — .00432).
(ii.) (.0$\dot{0}$441 + .351) by (.0072 — .00072).

(10.) Express as a decimal $(3\frac{101}{126} + 4\frac{5}{6} - 9\frac{7}{9} + 5\frac{3}{7}) \div (5\frac{1}{7} - 4\frac{9}{14} + 3\frac{1}{3})$

(11.) Find the difference between .3 of $\frac{5\frac{1}{2}}{6.6}$ yards and $\frac{1}{5}$ of $\frac{8.4}{2\frac{3}{5}}$ of an English ell.

12.) If 36 cwt. 0 qrs. 21 lbs. be carried 18 miles for £5 1 fl. 4 cents., how far should 15 cwt. 1 qr. 14 lbs. be carried for £2 7 fl. 8 cents?

(13.) A merchant buys 17.5 gallons of spirit at 7 flor. 25 mils. per gallon; what quantity of water must he add to it, so that by selling the mixture at 14s. per gallon, he may gain £1.725.

(14.) A ship leaves a port at the rate of 8 miles per hour; after 3 hours, a boat starts in pursuit at 10 miles an hour; and after another hour a second starts at 12½ miles per hour. When will they respectively reach the ship?

(15.) A person possessing 5/9 of a share in a coal field, sold .75 of his share for £2820, and lost £10 per cent. by the transaction; what was the original cost of his share?

(16.) A quadrant of the meridian, or ¼ the circumference of the globe = 100000565.268 French metres, and one metre = 39.37079 English inches; how many English yards are there in the whole circumference?

(17.) If 2 cubic inches of iron weigh as much as 15 of water, and a cubic foot of water weigh 62½ lbs.; what is the weight of a mass of iron containing 100 cubic feet?

(18.) Three persons whose estates are respectively worth £2000, £1510, and £1290 a year, buy 200 railway shares among them, each buying a number proportional to his estate; how many shares does each buy?

(19.) An agent charges 4¼ per cent. for commission and risk of bad debts; his sales in a year amount to £14,780, and his losses to £230; what is his net income?

(20.) What is the interest on £256, from May 7 till Aug. 12, at 4½ per cent.?

(21.) Find the amount of £850 in 3 years at 6 per cent. compound interest.

(22.) In 1851 the population of Liverpool was 375,955, and of Manchester 316,213; in 1861 the respective populations were 443,938 and 357,979. Find their respective rates of increase.

(23.) A debt which is due in 5 months is paid ⅓ in 3 months, ¼ in 4 months, ⅕ in 6 months; when will the remainder be payable?

(24.) Goods were purchased for £96, half was sold at a gain of 12 per cent, for what must the rest be sold to gain 16 per cent. on the whole?

(25.) (i.) When the Three per Cents. are at 92⅝, what must be given for £533 6s. 8d. stock?

(ii.) What rate of interest will be obtained by purchasing Three per Cents. at 91¼?

(26.) If a man sell a watch for £19, and lose by so doing £5 per cent.; what ought he to sell it for to gain £25 per cent.?

(27.) If 30 yards of cloth are worth $9\frac{1}{2}$ bushels of corn, and 13 bushels are equivalent to $\frac{16}{17}$ of £7 10s.; what is the value of 65 yards?

(28.) There are two squares, the side of one is 225 yards, and of the other 120 yards. Required the side of a square which shall be equal to the two together.

(29.) Find the area of a path 6 feet wide, which surrounds a court 30 yards long and 24 yards broad.

(30.) The content of a solid block of stone 12 ft. 6 in. broad, and 3 ft. 9 in. thick, is 27 cub. yards 1 cub. ft. 810 cub. in. Required its length, and its price at 6d. per cubic foot.

Ex. 97.

(1.) An estate consists of 54 a. 2 r. 11 p. $28\frac{1}{2}$ yds. pasture, 137 a. 3 r. 29 p. 17 yds. arable, 3 a. 0 r. 33 p. $15\frac{1}{4}$ yds. orchard, and a fir plantation of 6 a. 3 r. 17 p. 30 yds. Find the whole extent of the property.

(2.) Find the cost of 72 sheets of wool, each containing 1 pack 12 stones 12 lbs., at 13d. per pound; each stone containing 16 lbs., and each pack 15 stones.

(3.) Reduce 49772628 square inches to acres.

(4.) Find the value of:—

(i.) $182\frac{3}{4}$ things at £1 8s. 6d. each.

(ii.) 8 cwt. 1 qr. 21 lbs. at £2 0s. 8d. per cwt.

(iii.) 12 a. 3 r. 19 p. at £5 10s. 6d. per acre.

(5.) What is the *nett weight* of 19 bales of wool, the *average gross weight* of each being 3 cwt. 3 qrs. 12 lbs.; draft, 2 lb. per bale; tare, 16 lb. per cwt.?

(6.) Simplify:—

$$\frac{3\frac{1}{4} + 1\frac{2}{5}}{5\frac{1}{3} - 4\frac{1}{4}} \text{ of } \frac{13\frac{8}{9} \text{ of } 15\frac{3}{5}}{5\frac{1}{12} \text{ of } 48} + \frac{3\frac{1}{4} - \frac{5}{6} \text{ of } \frac{4}{15}}{(21\frac{1}{5} + \frac{4}{5} - 4\frac{1}{3} \text{ of } 5) \times 6\frac{1}{10}}.$$

(7.) Find the value of:—

(i.) $\frac{2}{7}$ of a pound + $\frac{47}{147}$ of a guinea + $\frac{211}{840}$ of 6s. 8d. $\frac{5}{63}$ of a crown.

(ii.) $\frac{3}{4}$ of £17 + $\frac{7}{18}$ of 27s. — $1\frac{7}{9}$ of £$2\frac{1}{4}$.

(8.) Find the value of $37.\dot{2}9\dot{7} + 21.\dot{0}37\dot{6} + .1\dot{9}23\dot{4} - 12.7\dot{8} - \dot{4}23\dot{1}$.

(9.) Multiply $1.8\dot{3}$ by $3.\dot{6}\dot{3}$ and divide the product by $3.41\dot{6} \div 8.2$.

(10.) A gramme = 15.433488 grs. *troy*, find the *avoirdupois* weight of a quintal or 100 kilogrammes.

(11.) Find the value of .75 of 6s. 8d. + 1.84375 of 4s. + $3.978\dot{3}$ of 2s.; also of $1.\dot{8}\dot{1}$ of $5\frac{1}{2}$ days + 5.75 of a week — $\frac{3}{4}$ of $5\frac{3}{8}$ hours.

(12.) After paying income tax at 25 mils. per pound, a person has £498 2 flor. 25 mils.; what was his gross income?

(13.) Find the total amount of the following bill:—

Wine, 57 hectolit. 22 litres at 2 fr. 45 c. per litre.
Wheat, 17 „ 15 „ at 18 fr. 25 c. per hectolit.
Sugar, $110\frac{1}{2}$ kilog. at 1 fr. 8 c. per kilog.

(14.) A plate of gold 3 inches square and $\frac{1}{8}$ of an inch thick, is extended by hammering so as to cover a surface of 7 square yards; find its present thickness.

(15.) If $113\frac{7}{16}$ yards of damask cost £13 14s. $3\frac{201}{400}$d., how many yards can be purchased for $\frac{1}{4}$ of £$45\frac{27}{80}$?

(16.) How many yards of lining .75 yards wide, will be required to line 55,75 yards of stuff .875 yards wide?

(17.) If 200 feet of iron railing cost £5 14s. 8d. when the bars are one inch and one quarter square; what will be the cost of 1000 feet when the bars are only seven-eighths of an inch square?

(18.) A ship is wrecked, whose value is £4500, of which £2400 belonged to A., £1200 to B., £600 to C., and £300 to D. The insurance office pays them back £3750 of their loss; how much of this should each receive?

(19.) What is the amount of £120, from Jan. 7 to Sept. 12, 1864 (leap year), at 4 per cent.?

(20.) If £42 3s. 9d. be received for interest on £11250 for 1 month, what is the rate per cent. per annum?

(21.) A person lays by £130 a year, and receives interest at 5 per cent. *compound* interest. What is he worth at the end of 4 years?

(22.) In a school of 30 boys, there are 4 boys aged 15 years, 6 each 14.75 years; 4 each 12.875 years; 3 each 11.8 years; 2 each $10.\dot{9}$, and 11 whose united ages make 113.256 years. What was the average age of the boys?

(23.) I owe £100 in 30 days, £270 in 118 days, and £365 in 1 year and 110 days; when ought the whole to be paid?

(24.) A man mixes 20 gallons of ale, which has cost him 11d. per gallon, with 65 of beer, which has cost 5d. per gallon; at what price must he sell the mixture in order to gain 20 per cent. on the outlay?

(25.) At what price must Three per Cent. Consols be bought, including

$\frac{1}{8}$ per cent. brokerage, to produce 5 per cent. interest after deducting 7d. in the pound for income tax?

(26.) If a tradesman lose 4s. 3d. by selling an article for £1 9s. 9d., what is the loss per cent.? What should he sell it for to gain $12\frac{1}{2}$ per cent.?

(27.) How many francs in 3412 rupees; the course of exchange between Calcutta and London being $23\frac{1}{4}$d. per rupee; and between London and Paris, 25.60 francs per £1?

(28.) Find the square roots of 1498176 and .002601, and the cube roots of 3581577 and .0005832.

(29.) How many *cubic* feet of lead $\frac{1}{16}$ inch thick will be required to cover the sides and bottom of a cistern 10 feet long, 6 feet 6 inches wide, and 7 feet deep? What weight of water will the cistern hold, a cubic foot of water weighing 1000 oz.?

(30.) How long will it take to empty a reservoir of water 15 feet long, 10 ft. 2 in. wide, and 10 ft. deep, at the rate of 10 gallons a minute, a gallon containing $277\frac{3}{11}$ inches?

Ex. 98.

(1.) Reduce:—

(i.) 345600 farthings into threepences, florins, and pounds.

(ii.) 12345 English ells into (i.) yards; (ii.) mètres.

(iii.) 175 miles into (i.) yards, and (ii.) mètres.

(2.) (i.) Find the weight of 53 casks of sugar, each weighing 3 cwt. 1 qr. 14 lbs.

(ii.) How often will a wheel, 4 ft. 7 in. in circumference, turn round in $12\frac{1}{2}$ miles?

(3.) I have a board 72 inches long and 18 inches wide, but I wish to obtain another board, that shall be 24 inches wide, and be just equal to the above. How long must it be?

(4.) Find the total cost of the following goods, allowing £5 per cent. for cash payment:—

Sugar, 13 cwt. 2 qrs. 21 lbs., at 53s. 8d. per cwt.
Tea, 7 cwt. 1 qr. 18 lbs., at $4\frac{1}{8}$s. per lb.
Bacon, 15 cwt. 18 lbs., at 98s. per cwt.; and
Rice, 3 bags, each containing 182 lbs., at 38s. 4d. per cwt.

(5.) Bought 17 bags of cotton, at $9\frac{3}{8}$d. per lb., each 2 cwt. 2 qrs. 15 lbs., deducting 1 lb. per bag for draft, Liverpool tare 4 lbs. per

112 lbs.; what shall I gain or lose by selling them again at 9⅛d., Manchester tare 2½ lbs. per 100 lbs.?

(6.) Simplify:—

(i.) $\left(5\frac{14}{15} + 3\frac{19}{20} + 11\frac{44}{45}\right) - \left(2\frac{15}{16} - 3\frac{17}{18} + \frac{26}{27}\right) + \left(4\frac{3}{4} - \frac{5}{6} - \frac{11}{12} + \frac{14}{15}\right).$

(ii.) $\left(1\frac{7}{10} - \frac{11}{15} + \frac{13}{18} - \frac{5}{36}\right) \times \left(\frac{1}{3} + \frac{3}{7} + \frac{4}{15} - \frac{11}{33}\right) \div \left(\frac{4}{7} - 1\frac{2}{9} + 5\frac{7}{8} - \frac{3}{11}\right).$

(7.) Simplify;—

(i.) $\dfrac{.005}{\frac{1}{15} \text{ of } 11\frac{1}{7}} \text{ of } \dfrac{49\frac{1}{21}}{\frac{7}{4} \text{ of } 2.25} \div \left(\frac{1}{21} + \frac{1}{27}\right).$

(ii.) $\dfrac{.0289}{.1169} \times \dfrac{1.002}{.374} \times \dfrac{3.375}{.008} \times \dfrac{.0495}{3.06} \div \dfrac{.0018}{.00252}.$

(8.) Add together £896 5 flor. 4 c. 7 m.; £397 5 flor. 3 mils.; £47 6 c. 2 mils.; £3 8 mils.

(9.) Find the value of:—

$3\dot{7}.29\dot{8} + 17.\dot{0}63\dot{7} - 19.\dot{4}3\dot{2} + .68\dot{3} - 11.\dot{7}\dot{4}.$

(10.) A. does ⅔ of a piece of work in 4 hours, B. does ¾ of what remains in 1 hour, and C. finishes it in 20 minutes. How long would they have been in doing the whole, if they had worked together.

(11.) Reduce the following to decimals:

(i.) $\dfrac{1\frac{7}{8}}{4\frac{2}{7}} - \dfrac{1\frac{7}{10}}{4\frac{1}{4}} + \dfrac{\frac{4}{7}}{2\frac{1}{2}}.$

(ii.) $\dfrac{1\frac{1}{5} - 1\frac{1}{7} + 1\frac{1}{9}}{1\frac{4}{5} - 1\frac{5}{7} + 1\frac{2}{3}} \text{ of } 1\frac{1}{2} \times \dfrac{\left(\frac{1}{6} + 7\frac{1}{3}\right)}{8\frac{1}{3} - \frac{3}{8}}.$

(12.) If 4 men or 6 women do a piece of work in 10 days, in how many days will 2 men and 8 women finish the same quantity of work?

(13.) A watch which gains 10¼ minutes in 24 hours is set right on Monday, at 8 a.m. What time will it indicate at 20 minutes past 4 on Wednesday afternoon?

(14.) Find the value of:—

484 ells of Holland, at 2s. 9⅝d. per yard.
457 „ „ „ 3s. 1⅚d. „
581 „ „ „ 4s. 2⅔d. „
1 ell = 1.09 yards.

(15.) Find the value of the following:—

Claret, 37.125 hectolitres at 140 fr. 25 c. per hectolitre.
Cognac, 26.48 hectolitres at 122 fr. 72 c. per hectolitre.
Plums, 287.50 kilogrammes at 1 fr. 80 c. per kilogramme.

(16.) Find the G. C. M. of 13434, 28152, and 214944; also the L. C. M. of 702, 936, 1872, 2808, and 3510.

(17.) A tax of £530 is to be raised from 3 towns, the respective numbers of inhabitants of which are 2516, 3023, and 4272; how much should each pay?

(18.) If 3 qrs. 21 lbs. of plums cost £1.05, what will be the cost of 6 casks weighing on an average 3.75 cwt. each?

(19.) What is the cost of £10,000 Three per Cent. Stock at 93⅝, with ⅛ per cent. for brokerage? What is the rate of interest on the purchase money?

(20.) (i.) What is the interest on £319 0s. 6d. for 5¾ years, at 3¾ per cent.?

(ii.) What principal put to interest at 3½ per cent. will amount to £603 0s. 2¼d., in 3 years.

(21.) Find the present value of £700 due in 4 years at 5 per cent (i.) at simple, (ii.) at compound interest.

(22.) The population of Great Britain in 1851 was 21,121,967, and the increase during the previous half century had been 93.5 per cent. What was the population in 1801?

(23) How many hhds. of spirits, at £80⅞ per hhd., are equivalent to 7.5 pipes of wine, at £121.3125 per pipe?

(24.) Find the present value of £1 due 1 year hence at 1 per cent. per annum.

(25.) If a person buy £550 in 3 per Cent. Stock at £81⅛, brokerage ⅛ per cent., and sell the same for £90¾, and with the proceeds buy 3½ per Cent. Stock at £105⅞; does he gain or lose in income by his transactions?

(26.) By selling goods at 8 shillings, I lost 16 per cent.; at what rate should I sell them to gain 7 per cent.?

(27.) How long a time would it take a column of men, extending half a mile in length, to pass by, at the rate of 75 steps a minute, reckoning each step as 2 feet 8 inches?

(28.) Extract the square root of 128881 and .0051804409, and the cube root of 786330467.

(29.) The foot of a ladder, 40 feet long, is 10 feet from a house, and its top reaches the upper part of a circular window; when it is drawn away 15 feet, its top reaches the lower edge of the window. Required the diameter and circumference of the window.

(30.) Find the price of a cylindrical iron ring, 12 inches thick, the inner diameter of which is 48 inches, at 3¼d. per lb., supposing the iron to weigh 7650 oz. per cubic foot.

Ex. 99.

(1.) Calculate the following invoice:—

1 Case:—

			oz.	dwts.	grs.		£	s.	d.	
1	Silver Punch Bowl	wt.	24	10	4	at	0	6	5	per oz.
1	„ Tankard	„	15	7	6	„	0	5	10	„
1	„ Tea-pot and Lamp	„	35	8	9	„	0	5	7½	„
12	„ Plates	„	125	2	7	„	0	5	8¾	„
18	„ Spoons	„	41	0	16	„	0	6	1	„
1	„ Waiter	„	16	8	0	„	0	6	2½	„

Tin and Wood Case, 10s. 6d.

2 Bales:—

170	pieces Norwich Crape	... ...	„	2	7	8	per piece
204	„ Kendal Cotton	... ...	„	1	16	3	„
5	„ Lancashire Sheeting, 175⅓ yards	...	„	0	1	2¾	per yard
12	„ Manchester Velvet, 698⅕ „	...	„	0	7	8	„
18	„ Yorkshire Drab 537⅛ ells	...	„	0	3	4	per ell
20	„ „ Forrester, 1001 „	...	„	0	6	8	„

Charges:—

To Entry and Duty, 70s., Dock Charges, &c., 50s.

To Freight, Primage, and Bills of Lading, £24 13s. 6d.

Insurance on £2,000 at 25s. per cent.

Policy Duty, 3s. 6d. per cent.

Commission on Insurance, ½ per cent.

Commission, 2½ per cent.

(2.) Bought a bag of cotton at 14¾d. per lb., weighing 3 cwt. 1 qr. 22 lbs. gross, tare 1 lb. in 70, I sold it again at 17¼d. per lb. allowing tare 3 qrs. of a pound in 70 lbs.; what was my gain?

(3.) Find the value of:—

$$\text{(i.)}\ \left(\frac{1\frac{3}{7}}{3\frac{1}{7}} \times \text{£}3\ 17\text{s.}\right) + \left(\text{£}8\ 14\text{s.}\ 6\tfrac{3}{4}\text{d.} \div \frac{1\frac{2}{3}}{1\frac{5}{11}}\right).$$

(ii.) 2 qrs. 17 lbs. $1\frac{7}{30}$ oz. — $8\frac{5}{6}$ lbs. — $3\frac{9}{10}$ oz.

(4.) A. can do a piece of work in 3 days, B. can do thrice as much in 8 days, and C. 5 times as much in 12 days; in what time would they do the work together?

(5.) Find the value of the following logs of mahogany at 1s. 2¼d. per cubic foot:—

No. 1, 43 ft. 10 in. × 5 ft. 4 in. × 2 ft. 11 in.

No. 2, 39 „ 7 „ × 4 „ 10 „ × 3 „ 4 „

(6.) Find the value of $.\dot{4}2857\dot{1}$ of £5 + $\left(\frac{1}{15} \times \frac{1}{2\frac{4}{5}} \times \frac{1}{4} \text{ of } 32s.\right)$ + $\frac{4}{81}$ of 3d., and express the result as the decimal of £6 13s. 4d.

(7.) A person laid out £23.108 in the purchase of 163 yards of cloth, of which 72 cost £.121 a yard; what was the price per yard of the remainder?

(8.) If A. and B. can do a piece of work in $4\frac{1}{11}$ days, A. and C. in $4\frac{3}{8}$ days, and B. and C. in $4\frac{11}{13}$ days; find the time in which they can do the work (i.) collectively and (ii.) separately.

(9.) Reduce 1,000,000 cubic inches to cubic mètres; and if the content of a cube be 405224 cubic mètres, find the length of the side in feet.

(10.) A merchant failed in business for £5760; he paid three dividends:—the first, 3 florins 5 cents in the £; the second, 2 florins 7 cents; the third, 1 florin 76 mils. How much did he pay in all?

(11.) If, when wine is 80 guineas a pipe, 60 persons drinking for a certain time spend 20 guineas; what is the worth of a pipe when 15 persons more, spend 65 guineas in twice the time?

(12.) A. can do a piece of work in 15 days, B. in 18 days; they work conjointly for 3 days, when B. leaves, and A. is joined by C., and they finish the work in 5 days. In how many days could C. have done it by himself?

(13.) If $2.\dot{6}$ ounces of sugar be worth .1875s., what is the value of .675 of a ton?

(14.) Find the interest, at $4\frac{3}{4}$ per cent., on £924 12s. 8d., lent on the 18th of March and repaid on the 11th of August.

(15.) (i.) In what time will £1000 amount to £1140, at $3\frac{1}{2}$ per cent.? (ii.) Find the interest on £193 12s. at £11 18s. 6d. per cent.?

(16.) What principal put to compound interest at 3 per cent. will amount to £1200 in 4 years?

(17.) A tradesman used by mistake a weight of $15\frac{5}{8}$ oz. instead of a lb. weight; how much did he gain per cent.?

(18.) If 5 men, 4 women, and 3 boys, can complete a piece of work in $16\frac{2}{3}$ days; what time would 9 men, 12 women, and 16 boys take to do four times as much, the parts done by each in the same time being as the numbers 3, 2, and 1?

(19.) What is the present value of £500, due 11 months hence, at $5\frac{1}{4}$ per cent. per annum?

(20.) A person, who has £700 Three per Cent. Consols, sells part of his stock, and purchases a £500 Six per Cent. Turkish Bond, Consols being at 87½ and Turkish at 70. What will be the difference in his income?

(21.) Bought quills at 15s., which I sold at £1, and 4 months' credit; required the gain per cent.?

(22.) A man buys 27 sheep for £30, and sells 12 of them at a loss of 3 per cent.; at what price must he sell each of the remainder that he may gain 2½ per cent. on the whole purchase?

(23.) Amsterdam draws upon London for 3000 florins; how much sterling will satisfy this draft, exchange at 12½ florins per £1?

(24.) Find the cost of making, at 3s. 6d. per square yard, a circular gravel walk 9 feet wide, so as just to touch the corners of a square garden, each of whose sides is 100 yards.

(25.) What is the weight of a cone, the height of which is 40 feet, and the diameter of its base 30 feet, supposing the material to weigh 2.618 oz. per cubic foot; and find the expense of painting the convex surface at 3¼d. per foot?

Ex. 100.

(1.) If 1869 sovereigns weigh 40 lbs. troy, how many sovereigns will weigh 2 ounces avoirdupois?

(2.) The carpet which covers a room 25 ft. long and 21 ft. broad is worth £13 2s. 6d. What is the breadth of a room 29 ft. long, the carpeting of which costs £20 6s.?

(3.) A bill of £655 10s., drawn Jan. 11, at 8 months, is discounted April 14, at 4 per cent. Find the difference between the true discount and the banker's discount.

(4.) Find the increase of income by transferring £12,000 Stock from the 3½ per Cents. at 85, to the 5 per Cents. at 96.

(5.) James Smith buys of you the following articles, on which you allow him a discount of 2½ per cent.:—21¼ yards black silk at 3s. 9d. per yard; 18¼ yards ribbon at 1s. 1½d. per yard; 34¾ yards calico at 4¾d. per yard; 16¼ yards cambric at 3s. 7½d. per yard. Make out James Smith's bill.

(6.) A shopkeeper buys coffee at £5 9s. 6d. per cwt., and chicory at £2 3s. 6d. per cwt., and mixes them in the proportion of two of chicory to five of coffee; he retails the mixture at 1s. 2½d. per lb. What is his gain per cent.?

(7.) Goods are sold for £2 9s. 0d. at 22½ per cent. profit; what was the cost price?

(8.) Henry Fowler buys of you 15½ lbs. of currants at 4d. per lb.; 17¼ lbs. of Malaga raisins at 5½d. per lb.; 19¾ lbs. of figs at 6d. per lb.; 17 lbs. of rice at 3½d. per lb.; 8½ lbs. of pepper at 1s. 6d. per lb.; 3 loaves of sugar, each weighing 32½ lbs., at 8½d, per lb.; 13 oz. of cloves at 9d. per oz. Make out Henry Fowler's bill, allowing a discount of 4¼ per cent.

(9.) If a person owes me £600, of which £300 is payable in 4 months and £300 in 6 months, what is the *true* present value, the rate of interest being 5½ per cent.?

(10.) Find a mean proportional between 2½ and 15⅝; also between .023 and .207.

(11.) A merchant buys 24 cwt. of sugar, at £2 11s. 4d. per cwt.; and sells ⅓ of it at 6½d. per lb.; ¼ at 6d., and the rest at 5½d.; what is his gain per cent.?

(12.) Bought 24 pipes of wine at £50 per pipe; part of the wine being lost, I sold the remainder at 20 per cent. profit, and by so doing realised 80 per cent. on the original outlay. How many pipes were lost?

(13.) How many francs are equal to 6824 rupees; if 1 rupee = 23¼d., and £1 = 25.60 francs?

(14.) A dealer mixes 7 qrs. 2 bush. of barley at 32s. per quarter, with 30 bush. at 26s. per quarter, and sells the mixture at 31s. 6d. per quarter; what is gained per cent.?

(15.) A. and B. are partners in trade, for every £3 A. puts in, B. puts in £4, at the end of 4 months A. increases his capital by ⅓, while B. draws out ¾ of his. At the year's end there is a gain of £340; how must it be divided?

(16.) How many books $1.7\dot{2}$ inches thick, must be placed on each other to form a pile 4 ft. $8.8\dot{3}$ inches high?

(17.) If 2 bodies were allowed to fall to the ground at the same instant, from heights of 500 and 200 feet respectively; how long would one reach the ground before the other?

(18.) If a body fall from rest during 10 seconds; what would be the spaces passed through in the 3rd and 6th seconds?

(19.) The adjacent edges of a block of marble are $3.\dot{4}2857\dot{1}$, $5\ \dot{1}4285\dot{7}$ and $10.\dot{2}8571\dot{4}$ inches. Find (i.) the surface, in square inches, of each face; (ii.) the solid contents of the block.

(20.) A field is 10.51 chains in length, and 4.28 chains in breadth; how many acres, roods, and poles does it contain?

(21.) Determine the volume and surface of a square pyramid, the edge of whose base is 10 mètres, and whose perpendicular height is 20 mètres.

(22.) An elliptical reservoir is to be fenced round; its major and minor axes are 24 mètres and 18 mètres. What length of fencing will be required?

(23.) A pole 50 ft. long being set up in a street, reached a window 36 ft. high on one side; and by turning it over without removing the foot, it reached a window 28 ft. high on the other side. Find the width of the street (i.) in feet, and (ii.) in mètres.

(24.) A cistern is 10 ft. long, 3 ft. 3 in. wide, and 3 ft. 9 in. deep. Find how much water it will hold, when filled within 2 inches of the top, in gallons and kilolitres.

(25.) A sphere of silver, whose radius is $1\frac{1}{4}$ in., is beaten out so as to form a circular plate $\frac{3}{10}$ in. thick; determine the diameter of the plate correctly to the third place of decimals.

(26.) A plantation, 16 mètres wide throughout, encloses a square field; each outer edge of the plantation measures 252 mètres in length. Find the number of hectares in the plantation, and also in the enclosed field; and express the same in acres, &c.

(27.) Find the depth of a cubical cistern which will contain the same quantity of water as another 10 feet long, 7 ft. 9.75 in. wide, and 3 ft. 1.5 in. deep.

(28.) The surface of a bowling green 112 mètres long and 81 mètres wide was raised $\frac{7}{36}$ of a mètre, by the earth taken out to form a pond 21 mètres square being spread evenly over it. How deep was the pond?

(29.) In a school-room 30 ft. long and 20 ft. broad, how far is it across the floor from one corner to the opposite one?

(30.) A merchant commenced trading with a capital of £10,000; he gained £10,000 every five years in the China trade, and £10,000 every 8 years in the American trade; he spent £10,000 every $2\frac{1}{2}$ years; in how many years did he become insolvent?

ARITHMETICAL TABLES.

MONEY.

1.—CURRENT COINAGE.

4 Farthings = 1 Penny (*d.*)
48 „ = 12 „ = 1 Shilling (*s.*)
960 „ = 240 „ = 20 „ = 1 Pound (£)

FARTHINGS TABLE.

Farthings		Pence	Farthings		Pence	Farthings		Pence	Farthings		Pence
4	are	1	16	are	4	28	are	7	40	are	10
8	...	2	20	...	5	32	...	8	44	...	11
12	...	3	24	...	6	36	...	9	48	...	12

PENCE TABLE.

d.		s.	d.	d.		s.	d.	d.		s.	d.	d.		s.	d.
12	are	1	0	48	are	4	0	84	are	7	0	120	are	10	0
20	...	1	8	50	...	4	2	90	...	7	6	130	...	10	10
24	...	2	0	60	...	5	0	96	...	8	0	132	...	11	0
30	...	2	6	70	...	5	10	100	...	8	4	140	...	11	8
36	...	3	0	72	...	6	0	108	...	9	0	144	...	12	0
40	...	3	4	80	...	6	8	110	...	9	2	240	...	20	0

SHILLINGS TABLE.

s.		£	s.	s.		£	s.	s.		£	s.	s.		£	s.
20	are	1	0	70	are	3	10	120	are	6	0	170	are	8	10
30	...	1	10	80	...	4	0	130	...	6	10	180	...	9	0
40	...	2	0	90	...	4	10	140	...	7	0	190	...	9	10
50	...	2	10	100	...	5	0	150	...	7	10	200	...	10	0
60	...	3	0	110	...	5	10	160	...	8	0	500	...	25	0

2.—DECIMAL COINAGE.

10 Mils (*m.*) = 1 Cent. (*c.*)
100 „ = 10 „ = 1 Florin (*fl.*)
1000 „ = 100 „ = 10 „ = 1 Pound (£)

WEIGHTS AND MEASURES.

Unit of	Imperial System.	Metric System.
Length ...	YARD ...	METRE, about $1\frac{1}{10}$ yards
Surface ...	SQUARE YARD	ARE, about 120 square yards
Solidity ...	CUBIC YARD	STERE, about 35½ cubic feet
Weight ...	POUND ...	GRAMME, about 15½ grains
Capacity ...	GALLON ...	LITRE, about 1¾ pints

IMPERIAL SYSTEM.

LENGTH.

1.—Long, or Lineal Measure.

12 Inches = 1 Foot (*ft.*)
36 „ = 3 „ = 1 Yard (*yd.*)
198 „ = 16½ „ = 5½ „ = 1 Pole, Rod, or Perch.
7920 „ = 660 „ = 220 „ = 40 „ = 1 Furlong (*fur.*)
63360 „ = 5280 „ = 1760 „ = 320 „ = 8 „ = 1 Mile.

2.—Cloth Measure.

The *Yard* in Cloth Measure is the same as in Long Measure, but differs in its divisions and subdivisions.

2¼ Inches = 1 Nail (*nl.*)
9 „ = 4 „ = 1 Quarter (*qr.*)
36 „ = 16 „ = 4 „ = 1 Yard (*yd.*)

SURFACE.

Square, or Land Measure.

144 Sq. In. = 1 Sq. Ft.
9 „ = 1 Sq. Yard.
272¼ „ = 30¼ „ = 1 Sq. Pole (*p.*).
9890 „ = 1210 „ = 40 „ = 1 Rood. (*r.*).
43560 „ = 4840 „ = 160 „ = 4 „ = 1 Acre (*a.*)

SOLIDITY.

Cubic Measure.

1728 Cubic Inches = 1 Cubic Foot.
= 27 „ = 1 Cubic Yard.

WEIGHT.

1.—Troy Weight.

24 Grains = 1 Pennyweight (*dwt.*).
480 „ = 20 „ = 1 Ounce (*oz.*).
5760 „ = 240 „ = 12 „ = 1 Pound (*lb.*).

2.—Avoirdupois Weight.

13 Drs. = 1 Oz.
256 „ = 16 „ = 1 Pound (*lb.*)
448 „ = 28 „ = 1 Qr
1792 „ = 112 „ = 4 „ = 1 Cwt.
35840 „ = 2240 „ = 80 „ = 20 „ = 1 Ton.

3.—New Apothecaries' Weight.

437.5 Troy Grains = 1 Ounce (*oz.*)
7000 „ = 16 „ = 1 Pound (*lb.*)

4.—OLD APOTHECARIES' WEIGHT.

20 Grains = 1 Scruple (℈).
60 ,, = 3 ,, = 1 Dram (ʒ).
480 ,, = 24 ,, = 8 ,, = 1 Ounce (℥).
5760 ,, = 288 ,, = 96 ,, = 12 ,, = 1 Pound (*lb.*)

In Old Apothecaries' Weight the ***pound*** and ***ounce*** were the same as in Troy Weight; in New Apothecaries' Weight they are the same as in Avoirdupois Weight.

CAPACITY.

2 Pints (*p.*).= 1 Quart (*qt.*).
8 ,, = 4 ,, = 1 Gallon (*gal.*).
16 ,, = 8 ,, = 2 ,, = 1 Peck (*pk.*).
64 ,, = 32 ,, = 8 ,, = 4 ,, = 1 Bushel (*bus.*)
512 ,, = 256 ,, = 64 ,, = 32 ,, = 8 ,, = 1 Quarter.

TIME.

60 Seconds = 1 Minute (*min.*).
60 ,, = 1 Hour (*hr.*).
1440 ,, = 24 ,, = 1 Day (*d.*).
8766 ,, 365¼ ,, = 1 Julian Year.

METRIC SYSTEM.

By Act of Parliament, passed 1864, the following Weights and Measures are permitted to be used in any contract or dealings.

LENGTH.

Metric Denominations and Values.		Equivalents in British Denominations.			
	Metres.	Miles.	Yards.	Feet.	Inches.
Myriametre	10,000	6	376	0	11.9
Kilometre	1,000		1,093	1	10.79
Hectomètre	100		109	1	1.079
Decametre	10		10	2	9.7079
METRE	1		1	0	3.3708
Decimetre	$\frac{1}{10}$				3.9371
Centimetre	$\frac{1}{100}$				0.3937
Millimetre	$\frac{1}{1000}$				0.0394

A Mètre is one ten-millionth of the distance from the pole to the equator.

SURFACE.

Metric Denominations and Values.		Equivalents in British Denominations.	
	Square Metres.	Acres.	Sq. Yards.
Hectare, *i.e.* 100 Ares	10,000	2	2280.3326
Decare, *i.e.* 10 Ares	1,000		1196.0333
ARE	100		119.6033
Centiare, *i.e.* $\frac{1}{100}$ Are	1		1.1960

CAPACITY

Metric Denominations and Values.		Equivalents in British Denominations				
	Cubic Metres.	Bush.	Pk.	Gal.	Qt.	Pints.
Kilolitre, *i.e.* 1000 Litres	1	27	2	0	0	0.77
Hectolitre, *i.e.* 100 Litres	$\frac{1}{10}$	2	3	0	0	0.077
Decalitre, *i.e.* 10 Litres ...	$\frac{1}{100}$		1	0	0	1.6077
LITRE	$\frac{1}{1000}$					1.76077
Decilitre, *i.e.* $\frac{1}{10}$ Litre ...	$\frac{1}{10000}$					0.176077
Centilitre, *i.e.* $\frac{1}{100}$ Litre	$\frac{1}{100000}$					0.0176077

WEIGHT.

Metric Denominations and Values.		Equivalents in British Denominations.				
	Grammes.	Cwt.	Qrs.	lb.	Oz.	Drams.
Millier	1,000,000	19	2	20	9	15.04
Quintal	100,000	1	3	24	7	6.304
Myriagramme ...	10,000			22	0	11.8304
Kilogramme ...	1,000			2	3	4.3830
Hectogramme ...	100				3	8.4383
Decagramme ...	10					5.6438
GRAMME	1					0.56438
Decigramme ...	$\frac{1}{10}$					0.056438
Centigramme ...	$\frac{1}{100}$					0.0056438
Milligramme ...	$\frac{1}{1000}$					0.00056438

PRACTICE TABLES.

ALIQUOT PARTS OF A

Pound.		*Shilling.*	*Cwt.*
10s. 0d. is $\frac{1}{2}$	1 8 is $\frac{1}{12}$	6d. is $\frac{1}{2}$	2 qrs. is $\frac{1}{2}$
6 8 ... $\frac{1}{3}$	1 4 ... $\frac{1}{15}$	4 ... $\frac{1}{3}$	1 qr. ... $\frac{1}{4}$
5 0 ... $\frac{1}{4}$	1 3 ... $\frac{1}{16}$	3 ... $\frac{1}{4}$	16 lbs.... $\frac{1}{7}$
4 0 ... $\frac{1}{5}$	1 0 ... $\frac{1}{20}$	2 is $\frac{1}{6}$	14 ... $\frac{1}{8}$
3 4 ... $\frac{1}{6}$	0 8 ... $\frac{1}{30}$	1½ ... $\frac{1}{8}$	8 ... $\frac{1}{14}$
2 6 ... $\frac{1}{8}$	0 6 ... $\frac{1}{40}$	1 ... $\frac{1}{12}$	7 ... $\frac{1}{16}$
2 0 ... $\frac{1}{10}$			

PRICE OF A CWT. AT PER POUND.

per lb.	per cwt.	per lb.	per cwt.	per lb.	per cwt.
¼d. is	2s. 4d.	3d. is	28s. 0	8d. is	74 8d.
½ ...	4 8	4 ...	37 4	9 ...	84 0
¾ ...	7 0	5 ...	46 8	10 ...	93 4
1 ...	9 4	6 ...	56 0	11 ...	102 8
2 ...	18 8	7 ...	65 4	12 ...	112 0

THE END.

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